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INTERACTIVE LOGIC IN THE MIDDLE AGES

Abstract. Recently logic has shifted emphasis from static systems developed for purely theoretical reasons to dynamic systems designed for application to real world situations. The emphasis on the applied aspects of logic and reasoning means that logic has become a pragmatic tool, to be judged against the backdrop of a particular application. This shift in emphasis is, however, not new. A similar shift towards "interactive logic" occurred in the high Middle Ages. We provide a number of different examples of "interactive logic" in the Middle Ages, all species of the disputation game *obligatio*. These games display a recognition of the importance of interaction in logical contexts and the way that interactive logic differs from single-agent inference.

Keywords: disputation, interactive logic, *obligatio*.

1. Games and interaction in logic

The recent trend in logic has been to shift emphasis from static systems developed for purely theoretical reasons to dynamic systems designed for application to real world situations, such as modeling knowledge, belief, interaction, and reasoning in multi-agent systems. The emphasis on the applied aspects of logic and reasoning means that logic has become both *situational* and *plural*: There is no single logic "with principles of inference valid for all possible subject-matters" [21, p. 741], but instead a multiplicity of logics and rules of inference that are appropriate or suitable (we specifically avoid the use of the term 'valid' here) in some contexts,

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and inappropriate or unsuitable in others. This means that logic has become, with this shift in emphasis, a pragmatic tool, whose merits are to be judged always against the backdrop of a particular application.

This emphasis on the situational and applied aspects of logic and reasoning is relatively new in contemporary logic, but it was the dominant approach to logic by logicians in the high Middle Ages, especially in the period from the mid-13th century to the mid-14th. Medieval logic was not concerned so much with abstract logical systems valid for all subject matters and suited for dealing with, e.g., mathematical reasoning, but more so in techniques of reasoning that could be applied in real reasoning contexts, and thus which could vary from context to context. This pragmatic approach to logic was complimented with a strong interest in modeling dynamic, interactive systems, where reasoning is not an armchair process of a single agent but instead consists of rational interaction, that is, a dispute, debate, or dialogue between multiple agents, each of which have different knowledge and different roles in the disputation. Thus, logic can be seen as a type of game, or rather, a multiplicity of types of games, each with its own rules, background knowledge, winning conditions, and goals.

The idea that logic should be understood or viewed as a type of game has a much longer history than its recent fashionability:

> The links between logic and games go back a long way. If one thinks of a debate as a kind of game, then Aristotle already made the connection; his writings about syllogism are closely intertwined with his study of the aims and rules of debating. Aristotle's viewpoint survived into the common medieval name for logic: dialectics. In the mid twentieth century Charles Hamblin revived the link between dialogue and the rules of sound reasoning, soon after Paul Lorenzen had connected dialogue to constructive foundations of logic. [14]

The best witness of the interactive approach to logic in the Middle Ages is the development of disputations *de obligationibus*. In an *obligatio*, two agents, the Opponent and the Respondent, engage in a turn-based dialogue where the Opponent puts forward a proposition (or set of propositions) at each round, and the Respondent can either accept, deny, or doubt the proposition(s), in accord with certain rules that are fixed in advance. The name *obligationes* derives from the fact that one of the players is "obliged" to follow special rules of discourse. The first treatises on *obligationes* date from the first few decades of the 12th century, and this type of disputation continued to be studied and developed through

the beginning of the 15th century, spanning the entire height of the development of logic in the Middle Ages. Many different variants are discussed, and many different sets of rules for each type of variant, and examples of obligational-style reasoning can often be found in texts on *sophismata* (logical puzzles and paradoxes), where the disputation rules are used as a meta-logic for reasoning about the *sophismata* sentences.

In this paper, we provide an overview of the development of *obligationes* — not merely historically, but also logically, focusing on those aspects which are of interest to those working in the new interactive turn in contemporary logic. We discuss the roots of *obligationes* in Aristotle and their relationship to types of dialogue games that he discusses, and we also critically compare a number of different examples of each of the different types (species) of *obligationes* identified in the literature.

It's worth commenting briefly on what this paper is not. First, despite it's title, it's not intended to be a survey of all the possible ways that logic in the Middle Ages was interactive. This would require a book instead of an article.[1] Instead, we focus only on theories of *obligationes*, the best witnesses to the interactive turn in medieval logic, as these demonstrate many of the interesting aspects that arise when logic is transposed into an interactive setting.

Second, this paper is not a logical investigation. While we discuss many of the interesting logical properties of *obligationes*, we introduce no formal systems and prove no new theorems. We do summarize recent work on formal models of properties of *obligationes*, in the final section; for more information the reader is directed to [9, 35, 34].

2. The roots and development of *obligationes*

William of Ockham in his *Summa Logicae* (written c. 1323) says that the art of *disputatio de obligationibus*:

> [...] consists of this that in the beginning some proposition has to be posited, and then propositions have to be proposed as pleases the opponent, and to these the respondent has to answer by granting or denying or doubting or distinguishing. When these answers are given, the opponent, when it pleases him, has to say: "time is finished". This

[1] Though one article which does discuss disputation more generally is [1].

> is, the time of the obligation is finished. And then it is seen whether the respondent has answered well or not.[2] [40, p. 67]

The *obligatio* continues until the Opponent calls "*Cedat tempus*" ("Time's up"), whereupon the responses of the Respondent are analysed with respect to the rules the Respondent was supposed to follow, to determine whether the Respondent has responded well or badly. While every author had his own idiosyncrasies with respect to the specific rules and constraints on the disputations, Ockham's definition here quoted is suitably generic; nearly every person who wrote on *obligationes* would subscribe to such a definition (with the caveat that not every author recognizes the fourth response, *distinguo*[3]).

While many of the earliest texts on *obligationes* are anonymous, and their exact dating, or their relative dating with respect to each other, cannot be positively established, it is clear that this genre saw its birth in the first decades of the 13th century [37]. In the following two centuries, scores of treatises were written on the topic, both by anonymous authors and by some of the leading logical lights of the time, including William of Sherwood (1190–1249), Nicholas of Paris (fl. 1250), Walter Burley (c. 1275–1344), Roger Swyneshed (d. 1365), Richard Kilvington (d. 1361), William Ockham (c. 1285–1347), Albert of Saxony (c. 1320–1390), John of Wesel (1340/50s), Robert Fland (c. 1350), John of Holland (1360s), Richard Brinkley (fl. 1365–1370), Richard Lavenham (d. 1399), Ralph Strode (d. 1387), Peter of Ailly (1351–1420), Peter of Candia (late 14th C), Peter of Mantua (d. 1399), Paul of Venice (c. 1369–1429), and Paul of Pergola (d. 1455).

Walter Burley in his canonical treatise *De obligationibus*, written around 1302 when he was a master of arts at the University of Oxford, defines the general goal of an *obligatio* as follows:

> The opponent's job is to use language in a way that makes the respondent grant impossible things that he need not grant because of the *positum*. The respondent's job, on the other hand, is to maintain

[2] "*Et consistit ars ista in hoc quod in principio debet aliqua propositio poni, deinde debent propositiones proponi secundum quod placet opponenti, ad quas debet respondens respondere concedendo vel negando vel dubitando vel distinguendo. Quibus responsionibus datis debet opponens, quando sibi placet, dicere: cedat tempus. Hoc est, cesset tempus obligationis. Et tunc videndum est an respondens bene responderit vel non.*" [38, p. 736]

[3] Drawing distinctions in a disputation is recognized as a legitimate move by Aristotle [2, Book VIII, ch. 7, 160a26–28].

> the *positum* in such a way that any impossibility seems to follow not because of him but rather because of the *positum*.[4] [6, p. 370]

Thus, it is clear that for Burley the goal of an *obligatio* is to show the consistency of a formula or set of formulas,[5] rather than to show the logical truth or validity of a formula; we return to this below when we discuss what types of games these obligation games are.

Burley's description of the goal of *obligationes* is noteworthy for being a nearly exact quotation from Book VIII, chapter 4 of the *Topics*.[6] In Book VIII of the *Topics*, Aristotle distinguishes three types of disputations: disputations for teaching and learning (*didactic*), disputations for competitive purposes (*eristic*), and disputations for the sake of practice and experiment (*dialectic*).[7] A dialectical disputation takes place between two participants who are working together to discover the truth of some matter; it is a cooperative game. A didactic disputation is also cooperative, but the participants are in different positions with respect to the truth being sought; one, the teacher, is trying to lead the other, the student, to some knowledge that the teacher has but the student does not. An eristic or sophistical disputation, however, is not cooperative, but strategic. In a sophistical disputation, the goal is to win,[8] not to discover truth, and any trick that can be used to reach this goal is acceptable.

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[4] "*Opus opponentis est sic inducere orationem ut faciat respondentem concedere impossibilia quae propter positum non sunt necessaria concedere. Opus autem respondentis est sic sustinere positum ut propter ipsum non videatur aliquod impossibile sequi, sed magis propter positum. Igitur intentio opponentis et respondentis circa enuntiabile versatur ad quod respondens est obligatus.*" [5, p. 34]

[5] Cf. [40, p. 70]. Such a view of the goal of *obligationes* was also espoused by Boethius de Dacia in his questions on the *Topics*, written between 1270 and 1276 [40, pp. 62, 65]. The importance of demonstrating mutual consistency of a set of propositions to the Aristotelian aims of science is discussed by Kakkuri-Knuuttila [16, pp. 241–42].

[6] This is also noted by Ekenberg [10, p. 25].

[7] This can be contrasted with ch. II of *On Sophistical Refutations*, where Aristotle says that "Of arguments used in discussion there are four classes: didactic, dialectical, examinational, and contentious arguments" [2, p. 279]. Examination-arguments are demonstrative arguments, which type are discussed in the *Prior Analytics*. Didactic and dialectical arguments are covered in the previous books of the *Topics*, and contentious arguments are the focus of *On Sophistical Refutations*.

[8] *Ad gloriam vel ad victoriam* — "for glory or for victory" — in Aristotle.

As Yrjönsuuri notes, the dialectical type of disputation "can be modernized into a question-answer game with two players: one asking the questions and the other answering them" [40, p. 60]. The roles of the two players, questioner and answerer, Aristotle describes as follows:

> With regard to the giving of answers, we must first define what is the business of a good answerer, as of a good questioner. The business of the questioner is so to develop the argument as to make the answerer utter the most implausible of the necessary consequences of his thesis; while that of the answerer is to make it appear that it is not he who is responsible for the impossibility or paradox, but only his thesis.[9]
>
> [2, p. 268]

In light of this, it is curious that Braakhuis says, speaking of Nicholas of Paris's *Obligationes*, that this is "the only text that we know of to date in which the standard approach to obligational disputation is linked with the Aristotelian theory of disputation presented in the *Topica*" [3, p. 159].[10] While other authors may not have mentioned Aristotle's name (in fact, most of them do not generally reference him explicitly), there are many indications that show that the medieval authors were interested in developing *obligationes* in the tradition of disputations as described in the *Topics* — not only Burley's quote above, but also in discussions concerning what type of disputations *obligationes* are. Nicholas of Paris, for example, says that with respect to their mode of operation, *obligationes* are a type of dialectical disputation, but with respect to the intended aim (namely, to trap the Respondent), they are a type of sophistical disputation [3, pp. 171, 1–11]. It is also likely that Aristotle is behind the worries of the anonymous author of *Obligationes Parisienses* when he wonders "whether to doubt is able to be an obligation or not".[11] In the *Topics*, Aristotle states that "a dialectical proposition must be of a form to which it is possible to reply 'yes' or 'no' " [2, p. 266]; in the species of *obligatio* called *dubitatio*, however, the response to the initial proposition

[9] On the relation of question-answer dialogues in Aristotle and the *dialegesthai* of Plato, see [16].

[10] He also says: "It is striking that [Nicholas of Paris] discusses the nature of obligational disputation so explicitly in relation to dialectical disputation as it is presented in the *Topica*. As yet, the only text in which such a relation is known to have been discussed is Boethius of Dacia's *Questiones super librum Topicorum*" [3, p. 158], though Yrjönsuuri has argued that Boethius's obligational theory is not canonical [40].

[11] "*Utrum 'dubitatur' sit obligatio annon.*" [7, p. 43]

is "I doubt it" (*dubio*), which is not one of the replies offered by Aristotle to a dialectical premise. (For a further discussion of this point, see [37, §2.2, §3].) Thus, the fact that Aristotle is often not mentioned explicitly is no argument against locating the roots of *obligationes* firmly in the Aristotelian tradition. The words of the Philosopher were assumed to be common knowledge among the educated, thus leaving little need for explicit attribution of his ideas to him (see also [33, p. 355]).

3. Definitions of technical terms

While medieval logicians worked with the same types of concepts that modern logicians do (e.g., truth, proposition, implication, equivalence, etc.), they often used different vocabulary. In this section we gloss some of these terms, which appear below both in translations of Latin texts and in the discussion and commentary on the texts.

consistens 'consistent' (said of a single proposition). A consistent proposition is one which is satisfiable, or not self-contradictory.

convertibile 'convertible'. If a proposition φ can be converted into a proposition ψ, then φ implies ψ. If two propositions are convertible with each other, they are (logically) equivalent.

enuntiabile 'enuntiable', something that can be be uttered or stated. Translated by Kretzmann and Stump as 'statable' in [6]. An enuntiable is a proposition.

(im)pertinens '(im)pertinent, (ir)relevant'. This technical term is defined differently by different authors; we give each author's definition when we discuss different variants.

repugnans 'repugnant'. A proposition φ is repugnant to a proposition ψ iff φ and ψ are inconsistent with each other.

significatio 'signification'. The signification of a proposition or a term is, roughly, its meaning.

4. The species of *obligatio*

The species of *obligatio* are differentiated by the rules which bind the Respondent's actions in the disputation. Many authors divide their rules into two types. On the one hand there are the general (or, principal, constitutive (*de esse*)) rules; these are often the rules which hold good

no matter what type of *obligatio* is in effect. On the other hand, there are the specific (or useful) rules, which are understood as giving strategic advice, rather than binding instructions, to the Respondent. Individual rules can be placed in different categories by different authors; for example, Nicholas of Paris's constitutive rule that "every response must be directed to the same instant" [3, p. 161][12] is merely a useful rule in Burley.

Walter Burley's general rules [6, p. 375] are representative of the standard view of *obligationes*; versions of these rules show up in almost every text.

> In connection with every obligation, three general rules are laid down, namely,
>
> [1] Everything following from an *obligatum* must be granted (where '*obligatum*' is interpreted as what has been granted or what must necessarily be granted);
>
> likewise,
>
> [2] Everything incompatible with the *obligatum* must be denied;
>
> likewise,
>
> [3] One must reply to what is irrelevant in accordance with its own quality.[13]

where a proposition is *irrelevant* or *impertinent*, for Burley, if neither it nor its negation follows from the set of propositions which have already been conceded (which includes the negations of propositions which have been denied).

While these general rules stayed the same in almost all versions of *obligationes*, the definition of relevance was often varied. Burley, and others in his tradition (which came to be known as the *responsio antiquo* 'old response') such as William of Sherwood, Ralph Strode, and Peter of Candia, computed the relevance of a proposition in terms of *all* of the Respondent's responses in the disputation so far. This was the standard definition of relevance until Roger Swyneshed offered a new definition in his *Obligationes*, written between 1330 and 1335. Swyneshed was followed in this redefinition by Robert Fland, Richard Lavenham, and

[12] "*Omnes responsiones retorquende sunt ad idem instans.*" [3, p. 177]

[13] "*In omni obligatione ponuntur tres regulae generales, scilicet, quod omne sequens ad obligatum est concedendum, intelligendo per obligatum, concessum vel necessario concedendum. Similiter, omne repugnans obligato est negandum. Similiter, ad impertinens respondendum est secundum sui qualitatem.*" [5, p. 42]

John of Wesel, among others.[14] In fact, modern scholars have classified various obligational theories on the basis of how they define irrelevant propositions — whether they take the static approach where propositions are evaluated with respect to only the *obligatum* or the dynamic approach where propositions are evaluated with respect to all of the Respondent's actions so far — and on the basis of how the Respondent is to respond to such irrelevant propositions — whether he should grant or deny them on the basis of their truth, or whether he should grant all of them, irrespective of their truth (see Table 1).[15] Two important consequences of these differing definitions are connected with the utility of the Opponent repeating a locution and the sensitivity of the Respondent's obligations to the order that the propositions are put forward. We discuss these further in the final section.

Type of relevance: Grounds for concession:	**Static**	**Dynamic**
All	Boethius	?
Only true	Swyneshed	Burley, Kilvington

Table 1. Classification in terms of irrelevant propositions

A second way in which the species of *obligatio* can be divided is into those in which the Respondent has an obligation *ad actum* (for an act) and those in which he has an obligation *ad habitum* (for a habit/disposition)[16]; there are three possible dispositions a Respondent may have towards a proposition: *concedere* 'to concede', *negare* 'to deny', and *dubitare* 'to doubt'. Both of these divisions are further divided into whether the object of the obligation is noncomplex (simple) or complex. Thus, on the basis of this division, Burley, along with many of the 13th C authors, identifies six different types or species of *obligatio*:

[14] Interestingly, this *nova responsio* of Swyneshed was not actually that new; about six decades earlier, a similar definition of relevance was offered by Boethius de Dacia [40, pp. 68, 63].

[15] Note that we have not placed all the authors discussed in this paper into the table, but instead have picked some representative authors for each category. Note also that this means that we do not currently know of any treatise on *obligationes* where a dynamic conception of relevance is adopted and yet all irrelevant propositions should be accepted, regardless of their truth value.

[16] Cf. [5, p. 34], [29, ¶30].

ad habitum

- Noncomplex
  - *impositio / institutio.*
- Complex
  - *positio.*
  - *depositio.*
  - *dubitatio.*

ad actum

- Noncomplex
  - *petitio.*
- Complex
  - *rei veritas / sit verum.*

Burley defines these six types in the following way:

> If [the obligation] obligates to an act and covers what is noncomplex, it is *petitio*. If it covers what is complex, it is *sit verum*. If it obligates to a disposition and covers what is noncomplex, it is *institutio*. If it covers what is complex, it obligates either to maintaining the complex as true, and then it is *positio*; or it obligates to to maintaining it as false, and then it is *depositio*; or to maintaining it as uncertain, and then it is *dubitatio*. And so there are six species of obligation.[17] [6, p. 370]

Later authors, from the second quarter of the 13th C on, in both the *responsio antiqua* and *responsio nova* traditions, tend to reduce the species of *obligatio* to three: *positio*, *depositio*, and *impositio*.[18] We discuss in the sections on *dubitatio*, *petitio*, and *sit verum* below how authors reduced these three to the other three, and whether these reductions are in fact correct.

Paul of Venice, in his early 15th-century *Tractatus de Obligationibus*, also gives a threefold division, made according to the types of replies that they allow, but it is nonstandard. His division is into *suppositio*,

[17] "*Si obliget ad actum et cadad super incomplexum, sic est 'petitio'. Si super complexum, sic est 'sit verum'. Si obliget ad habitum et cadat super incomplexum, sic est 'institutio'. Si super complexum, aut obligat ad habendum pro vero, et sic est 'positio', aut ad habendum pro falso, et sic est 'depositio', aut ad habendum pro dubio, et sic est 'dubitatio'. Et sic sunt sex species obligationis*" [5, pp. 34–35]. A similar division, though not stated nearly as precisely, is found in [7, pp. 27–28]. Nicholas of Paris's division is also not so explicit. In two different places he mentions five of the six; in the first place, he omits *petitio* and in the second, he omits *institutio* [3, p. 159].

[18] "*In tres tantum sunt species obligationis, videlicet, positio, impositio, depositio*" [27, ¶ 1]; "John cites Walter Burley by name as holding that there are six species of *obligationes*. In his reply, John cites Roger Swyneshed's view ("auctor in littera") that there are only three species: *positio*, *depositio*, and *impositio*" [29, p. 7] (see also [29, ¶¶ 19, 30]).

positio, and *depositio* [25, p. 37]. *Positio* and *depositio* are familiar, but *suppositio* is mentioned in no other text. In *suppositio*, the Opponent begins by making an assumption or *suppositio*, for example "I assume the Antichrist exists", and then the Respondent's obligations are tied to this assumption. This unique species of *obligatio* incorporates and formalizes a technique that is often found in earlier treatises, namely that of introducing a *casus*, an explicit partial world-description that is common knowledge between both players [41]. (The presence of a *casus* is sometimes appealed to when contemporary scholars argue that *obligationes* provide a framework for counterfactual reasoning. We comment on this analysis in §5 below.)

So, what are the rules that govern each of these different species, according to various authors? How does changing the rules affect the interactive aspects of the disputations?

4.1. *Positio*

Positio is the crown jewel of the *obligationes* regalia. It is the most prominently discussed, by both medieval and modern authors. We consider Burley's rules, which are typical of 13th century developments[19], and then contrast them with the slightly later developments of Richard Kilvington (still *responsio antiqua*, but introducing some changes in the rules), and Roger Swyneshed, indicative of the 14th C *responsio nova* tradition.

4.1.1. *Positio* according to Burley.

Burley defines *positio* as "a prefix to something statable [indicating that the statable thing] should be held to be true" [6, p. 378].[20] *Positio* itself can be divided into multiple types. The first division is into possible and impossible *positio*; both divisions are further divided as to whether the enuntiable is simple or complex, and then further as to whether the complex enuntiables are formed by conjunction ("conjoined *positio*") or disjunction ("indeterminate *positio*"). In any of these types of *positio*, it is also possible that a further stipulation is added, in which case the *positio* is called "dependent" [6, p. 378].

[19] With the caveat, thanks to Yrjönsuuri [40], that Boethius of Dacia does not completely fit in the Burleyan tradition.

[20] "*Positio, secundum quod hic sumitur, est praefixio enuntiabilis ad habendum pro vero.*" [5, p. 45]

The bulk of the discussion of *positio* focuses on possible *positio*, and we follow suit. The general rules governing possible *positio* are as follows:

RULE 4.1. Everything that is posited and put forward in the form of the positum during the time of the positio must be granted [6, p. 379].[21]

RULE 4.2. Everything that follows from the positum must be granted. Everything that follows from the positum either together with an already granted proposition (or propositions), or together with the opposite of a proposition (or the opposites of propositions) already correctly denied and known to be such, must be granted [6, p. 381].[22]

RULE 4.3. Everything incompatible with the positum must be denied. Likewise, everything incompatible with the positum together with an already granted proposition (or propositions), or together with the opposite of a proposition (or the opposites of propositions) already correctly denied and known to be such, must be denied [6, p. 381].[23]

These three rules are just specifications of the two general rules with which we opened the section. These rules, quite simple in nature and in number, exhaustively cover all of the possibilities that the Respondent may face in the course of the disputation. Burley also provides a number of what he calls "useful" rules, which are related not to playing the game, as the general rules are, but to playing the game *well*. They are not so much rules as they are descriptions or properties of disputations that are played according to the constitutive rules, or strategic advice to the Respondent.

Let us now give some examples of actual *positiones* (though with the actual propositions abstracted). The first, given in Figure 1, is fairly simple; variants of it show up in many of the 13th century treatises. Suppose that φ does not imply $\neg\psi$ and φ is known to be false. Since φ is satisfiable (because if it were not, then it would imply $\neg\psi$), the Respondent should admit it when it is put forward as a *positum*. In

[21] "*Omne positum, sub forma positi propositum, in tempore positionis, est concedendum.*" [5, p. 46]

[22] "*Omne sequens ex posito est concedendum. Omne sequens ex positio cum concesso vel cum concessis, vel cum opposito bene negati vel oppositis bene negatorum, scitum esse tale, est concedendum*". [5, p. 48]

[23] "*Omne repugnans posito est negandum. Omne repugnans posito cum concesso vel concessis, vel opposito bene negati vel oppositis bene negatorum, scitum esse tale, est negandum*". [5, p. 48]

	Opponent	**Respondent**
1	φ.	I admit it.
2	$\neg\varphi \vee \psi$.	I grant it.
3	ψ	I grant it.

Figure 1. A simple example.

	Opponent	**Respondent**
1	φ or φ must be granted.	I admit it.
2	φ must be granted.	I deny it.
3	φ follows from the positum and the opposite of something correctly denied	I grant it.
4	φ must be granted.	??

Figure 2. A more complex example.

the second round, Opponent asserts $\neg\varphi \vee \psi$. Now, either φ implies ψ, in which case the proposition follows from the *positum* and hence the Respondent should concede it, or ψ is independent of φ, and hence the proposition is irrelevant. In that case, we know that since φ is false, $\neg\varphi$ is true, so the disjunction is true, and true irrelevant propositions should be conceded. But then, ψ follows from the *positum* along with something correctly conceded, and hence when the Opponent asserts ψ, the Respondent must concede it too. This example shows how, given a *positum* which is false, but not contradictory, the Opponent can force the Respondent to concede any other proposition consistent with it. The fact that this is possible is one of Swyneshed's primary motivations for revising the standard rules. The next example, in Figure 2, is less trivial, and illustrates how complex higher-order reasoning about the Respondent's obligations can take place within an *obligatio* itself.

Assuming, again, that φ is satisfiable, since the entire complex *positum* ψ_1 ="φ or φ must be granted" is also satisfiable, then the Respondent should admit it. In the next round, the Opponent asserts ψ_2 ="φ must be granted"; this is either relevant or irrelevant. While ψ_2 is identical to one of the disjuncts of ψ_1, there is nothing which forces it to also be implied by the other. So, ψ_2 is neither a consequent of nor inconsistent with ψ_1, and hence it is irrelevant, and Respondent's response must be on the basis of its truth. None of the rules for *positio* obligate the Respondent to concede φ, thus, ψ_2 is false, and Respondent must deny it. In round three, Opponent merely points out the validity

of disjunctive syllogism; since this statement is also irrelevant, and true, the Respondent must admit it. But now, he's faced with a problem in round four. Since he has accepted that φ follows from ψ_1 along with the negation of ψ_2 (one of the disjuncts of ψ_1), he is now forced to grant φ, and hence he must grant that he must grant φ. But this is precisely what he denied in round 2. Whichever move he makes at this point, he will have responded badly. Burley's solution to this paradox is to say that the Respondent shouldn't have accepted the *positum* in the first place; even though it looked innocent, it in fact contained a hidden paradox. Many of Burley's (and other authors') examples are of this type, showing that apparently innocuous *posita* are actually problematic, showing the utility of the obligational framework for exposing possible contradictions. This again is connected with the question of whether the games should be viewed as cooperative or strategic.

Our first example above shows how, provided that the *positum* is consistent, the Opponent can force the Respondent to concede anything consistent. Further formal properties, following from the assumption of a consistent *positum* (which is the definition of possible *positio*), include that no disputation requires the Respondent to concede φ in one round and to concede $\neg\varphi$ in another round (or to concede φ in one round and to deny it in another); the set of formulas conceded, along with the negations of those denied, will always be a consistent set; yet, it may be that the Respondent has to give different answers to the same propositions put forward at different times. This last property is another motivation of Swyneshed's revised rules, which we turn to next; we also discuss this point in §5.

4.1.2. *Positio* according to Swyneshed.

Roger Swyneshed's *Obligationes* [26], written sometime between 1330 and 1335, represents a sharp shift in approach. He found a number of the consequences of Burley's system unpalatable, and introduced new definitions in order to block those consequences. In particular, as we noted above, he disliked the fact that the Opponent could force the Respondent to concede any proposition consistent with the *positum* (assuming the *positum* itself were consistent), and the fact that the Respondent could be forced to give different answers to the same proposition put forward at different times. Interestingly, both of these problematic (at least for Swyneshed) consequences can be blocked with a single simple change of definition. His rules for *positio* are effectively the same as Burley's

[26, ¶¶62, 67–69]; he merely changes the definition of relevance. On Swyneshed's definition

> Some of the propositions are pertinent to the *obligatum*, others are impertinent to the *obligatum*. And of those pertinent to the *obligatum*, some follow from the *obligatum*, and others are repugnant to the *obligatum*.[24]

Later, he turns this into an explicit definition:

> The second definition is this: A pertinent proposition is a non-obligated proposition which, however it may signify, because of the *obligatum* must be conceded or denied.[25]
> The third definition is this: An impertinent proposition is a non-obligated proposition, and because of the *obligatum* it must neither be conceded nor denied.[26]

Thus, a proposition is relevant if either it or its negation follows from the initial statement or its negation follows from the *positum*, and otherwise it is impertinent. Like Burley's *positio*, provided that the *positum* is consistent, Swyneshed's *positio* has the property that in no disputation will the Respondent be required to concede φ in one round and deny it (or concede $\neg\varphi$) in another round.

But how does this redefinition of pertinence block the unwanted consequences of Burley-style *positio*? It does so by changing the nature of the game from an essentially dynamic game, where the Respondent's actions may change his future obligations, to a static game, where all of the Respondent's actions could, in a sense, be analyzed at once — that is, the sequential order of the propositions put forward by the Opponent no longer contributes anything to the disputation. The Opponent could just as easily put forward all the propositions he desires at once, and the Respondent could pick and chose the order in which he wishes to respond to them. Thus, the trick used in *positio* in Figure 1, whereby ψ became relevant after the concession of $\neg\varphi\vee\psi$, does not happen here; if ψ was not relevant after round one, it would not become relevant after round two,

[24] "*Propositionum alia est pertinens obligato, alia est impertinens obligato. Et pertinentium obligato alia est sequens ex obligato, alia repgunans obligato*". [26, p. 251]

[25] "*Secunda definitio est haec: Propositio pertinens est propositio non obligata quae, qualitercumque significet, propter obligatum est concedenda vel neganda.*" [26, p. 252]

[26] "*Tertia definition est haec: Propositio impertinens est propositio non obligata, et propter obligatum nec est concedenda nec neganda.*" [26, p. 252]

	Opponent	Respondent
1	φ.	I admit it.
2	ψ.	I doubt it.
3	$\neg\varphi \vee \psi$.	I grant it.
4	ψ.	I doubt it.

Figure 3. A Swyneshedian *positio*.

and in fact would never become relevant. This also shows Swyneshed can block the possibility that Respondent's answers to a proposition will change.

Consider the *positio* in Figure 3, where φ is false but satisfiable, and the truth value of ψ is not known — a typical example of such a sentence is "The pope is in Rome" (he might be in Avignon), or "The King of France is sitting" (he might be running or sleeping or disputing). This example is like the *positio* in Figure 1, but Opponent asserts ψ in both round two and round four. At the second round, since ψ is irrelevant (on both Swyneshed and Burley's definition of relevance), Respondent should responded to it according to its quality. Since its truth value is unknown, then the Respondent must doubt the proposition. Consider the fourth round: on Burley's rules, ψ is now relevant, as we saw above, so Respondent must change his answer, and concede it. But on Swyneshed's definition of relevance, ψ is still irrelevant at round four, and thus Respondent need not change his answer. Thus, such a situation as the one allowed by Burley's rules will never happen under Swyneshed's definition of relevance.

Swyneshed's redefinition of pertinence may have solved the problems that he saw in the *responsio antiqua*, but this new version has unintuitive consequences of its own. As various people have pointed out (including [8, 28, 40]), because his definition of relevance does not adapt and change over the course of the disputation, it is possible for the set of sentences that the Respondent concedes (along with the negations of the ones he denies) to be inconsistent; a particular case that is discussed by Swyneshed himself is the possibility of conceding a conjunction while denying both conjuncts [26, ¶32] (note that this is a *consequence* of his rules, not a rule itself, contra [40, p. 73]).

This, in and of itself, is not so problematic, when you consider that the responses to the relevant propositions will always be consistent, and the responses to the irrelevant propositions will always be consistent, and

it is only the union of these responses that may be inconsistent. More problematic, and this is something that few modern commentators have noticed, is that by switching to a static definition of relevance, instead of the dynamic version used by the *responsio antiqua*, Swyneshed (and others, such as Boethius of Dacia [40, pp. 66–67]) loses any sense of *disputation*. If the order of the propositions no longer matters for the determination of the responses, then, as we noted above, there isn't any need for the Opponent to give the propositions one by one. In fact, there isn't really any need for the Opponent at all; once he's generated his list of propositions and designated one of them as the *positum*, then he can hand them over to the Respondent and leave; his work is done. The difference is, in a sense, between an exam proctor who hands out the tests and then does nothing further while the students fill in their answers, and one who gives an oral examination of the student, where the student's responses to previous questions are taken into account. Additionally, this lack of dynamicity trivializes Swyneshed's games. It is precisely because the relevance of propositions must be recalculated at each stage of the disputation that Burley-style *obligationes* are difficult to play.

4.1.3. *Positio* according to Kilvington.

The third type of *positio* that we consider comes from Richard Kilvington's *Sophismata*, written c. 1325 [17, 18]. This is a treatise on logical puzzles and paradoxes, many focusing on issues related to natural philosophy. However, in the treatment of the final *sophisma* in the text, "A is known by you"[27], where A is the either "God exists" (a necessary truth, which is necessarily known) or "Nothing granted by Socrates is known by you", where Socrates grants that "A is known by you", and nothing else. Kilvington introduces and uses obligational-style rules for resolving the question of whether the proposition "A is known by you" can be conceded, denied, or doubted. These rules have been reconstructed by Stump [31] and Spade [28], and discussed further in [41].

Because Kilvington does not give an explicit obligational theory apart from the specific application of *obligatio*-style reasoning to a particular *sophisma*, it is difficult to classify it precisely with respect to the species discussed above. While much of what he says is in the language and guise of *positio*, given the nature of the *positum*, his obligation rules actually bear more resemblance to *dubitatio* than to *positio*. For if A is

[27] *Sophisma* 48 [17, pp. 137–151], [18, pp. 134–145].

in fact the second possibility, namely that "Nothing granted by Socrates is known by you", then given that Socrates grants "A is known by you" and nothing else, then in order to maintain A, the Respondent must be in doubt concerning the status of A, despite the fact that Socrates grants that A is known by the Respondent. But this — the requirement to doubt a sentence which is known — is a feature unique to *dubitatio*. In particular, in *positio*, the Respondent is never required respond "I doubt it" to a proposition whose truth value he knows (for either the propositio is relevant, in which case he must concede or deny it as appropriate, or it is irrelevant, and since he knows its truth value, he must either conceded or deny it, again as appropriate).

Kilvington can be placed in the *responsio antiqua* as he follows Burley's dynamic definition of pertinence. However, he treats irrelevant propositions differently. Depending on how his views are reconstructed, he either redefines irrelevance in terms of what *would* follow from (or be repugnant to) the *positum*, and other propositions already conceded or the negations of those denied, *were* the *positum* true, or, alternatively, he changes the rule determining Respondent's responses to irrelevant propositions. Instead of Respondent conceding those which he knows to be true, denying those which he knows to be false, and doubting if he doesn't know, Kilvington's rule says that the Respondent must concede the proposition if it would be true were the *positum* true, and to deny it if it would be false were the *positum* true.

4.2. *Depositio*

Depositio is just like *positio*, except that the Respondent is obliged to deny or reject the initial proposition (the *depositum*). A *depositio* with *depositum* φ will be completely symmetric to a *positio* with $\neg\varphi$ as the *positum*. Nevertheless, early treatises on *obligationes*, such as that by Nicholas of Paris which dates from ca. 1230–1250, still treat *depositio* at some length. "In the section on *depositio* the question is raised as to whether, when a contingent true proposition is deposited, any (contingent) true proposition that is compossible with the *depositum* can be proved, in the same way that any (contingent) false proposition that is compossible with the *positum* can be proved or conceded when a contingent false proposition is posited" [3, pp. 156–157].[28] The use here

[28] "*Datur pro regula quod falso possibili posito de quolibet falso compossibili illi debet concedi ipsum esse.*" [3, p. 202]

of 'prove' is infelicitous; in a case like that which Braakhuis has just described, nothing at the propositional level is being *proved*. It is only meta-level properties, such as consistency and compossibility, that are proved or disproved in an obligational disputation.

4.3. *Dubitatio*

In *dubitatio*, the Respondent must *doubt* the statement that the Opponent puts forward (called the *dubitatum*). While *dubitatio* was discussed in 13th century texts, often at some length, later authors (both later medieval and modern authors) call *dubitatio* a trivial variant of *positio*, and thus spend little time discussing it. For example, Paul of Venice [25] reduces *dubitatio* to *positio* (in much the same way that he, and others, reduces *depositio* to *dubitatio*); Swyneshed, Lavenham, John of Wesel, Richard Brinkley [4], and John of Holland [15] do not mention *dubitatio* at all. However, as we have argued elsewhere [37, 34], such a trivializing view of *dubitatio* fails to recognize the higher-order aspects of the disputation, the mixing of both knowledge and truth, which result in a significantly more difficult type of game, which, contrary to Stump [33, p. 372, fn. 14], does not involve any type of three-valued logic; while there is a tripartite structure of the dispositions of the Respondent (i.e., that certain propositions must be doubted, certain must be denied, and certain others must be conceded), these dispositions should not be taken as assigning (epistemic) truth values to the propositions. Just as *positio* is only interesting when the *positum* is false, *dubitatio* is only interesting when the truth value of the *dubitatum* is known (whether it is true or false). Thus, part of the complexity of the game arises from the interaction between knowledge, truth, and the obligations of the Respondent, as the Respondent in many cases is required to respond *dubio* 'I doubt it' to propositions that he actually knows.

One of the earliest texts that discusses *dubitatio* at any length is the anonymous *Obligationes Parisienses* edited by de Rijk in [7]. This text, tentatively datable to the first or second decade of the 13th C [37, §2.1], is also one of the few that discusses the status of *dubitatio* as a legitimate distinct species of *obligationes* (see §2 above). The author gives two general rules, for propositions which are not equivalent with the *dubitatum*, and eight specific rules and facts which give more explicit rules governing the Respondent's actions. Because we have analysed these rules in detail elsewhere (along with the rules in William of Sherwood's treatise [39])

[37], we do not discuss them here, but instead look at one of the most extensive and comprehensive 13th-century discussions of *dubitatio*, found in Nicholas of Paris's treatise. Nicholas gives seven rules for *dubitatio* [3, pp. 72–76]. Many of these rules are similar to those for *positio* and *depositio*:

RULE 4.4. Just as in false *positio* it is impossible to put forward "a falsehood is put forward" nor in *depositio* "a falsehood is to be deposed", by the same reason it is impossible to doubt "a falsehood is doubted".[29]

RULE 4.5. Just as in *positio* a *positum* put forward in the form of the *positum*, and everything convertible to it in the time of positing is to be conceded and its opposite and things convertible with it is to be denied and just as in *depositio* a *depositum* put forward in the form of the *depositum*, with its convertibles, must be denied and its opposite with things convertible with it must be conceded; so in *dubitatio* for a *dubitatum* put forward in the form of *dubitatum* and for its convertibles and moreover for the opposite of the *dubitatum* with its convertibles must be answered "prove!"[30]

RULE 4.6. For everything irrelevant to the *dubitatum* the response must be according to its quality.[31,32]

RULE 4.7. All the responses must be directed to the same instant.[33]

However, the rules that govern relevant propositions have interesting consequences. He says:

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[29] "*Sicut in falsa positione non potest poni* falsum poni *nec in depositione* falsum deponi, *ita nec in dubitatione potest dubitari falsum dubitari.*" [3, p. 223]

[30] "*Sicut in positione positum sub forma positi propositum et omne convertibile cum illo in tempore positionis est concedendum et suum oppositum cum suo convertibili negandum, et sicut in depositione depositum sub forma depositi propositum cum suo convertibili negandum et suum oppositum cum suo convertibili concedendum, ita in dubitatione ad dubitatum sub forma dubitati propositum et ad suum convertibile et preterea ad oppositum dubitati cum suo convertibili respondendum est 'proba'.*" [3, p. 223]

[31] "*Ad omne impertinens dubitato respondendum est secundum sui qualitatem.*" [3, p. 225]

[32] Note that while Nicholas's definition of relevance [3, p. 179] taken strictly mirrors Swyneshed's, it is clear from the succeeding rules that his notion of relevance is the same as Burley's and other 13th C authors.

[33] "*Omnes responsiones retorquende sunt ad idem instans.*" [3, p. 227]

	Opponent	Respondent
1	'Socrates is white'	I doubt it.
2	'Socrates is white'	I doubt it.
3	'Socrates is pale/fair'	I doubt it.
4	'Socrates is black'	I concede it.
5	'It is false that Socrates is white'	I concede it.
6	*Cedat tempus*	

Figure 4. *Dubitatio* according to Nicholas of Paris.

Rule 4.8. For everything antecedent to the *dubitatum* the response must be "false" or "prove!" and never "true".[34]

Rule 4.9. For everything consequent to the *dubitatum* it is possible to reply "it is true" or "prove" and never "it is false".[35]

Here we see another cause of the complexity of *dubitatio*, and how it cannot simply be reduced to *positio* or *depositio*: This is because the rules, unlike the rules for the other types, are not deterministic. Whereas there is always a unique correct response for Respondent in *positio* (in both the *responsio antiqua* and *nova*), here, the rules give Respondent a range of choices. This non-determinacy means that there is a plurality of ways that Respondent may act, and still be disputing according to the rules, a feature which no other version of *obligatio* has. However, this feature of *dubitatio* seems not to have been noticed by later authors who insisted that *dubitatio* could be reduced to *positio*.

The second to the last rule he gives is the most curious:

Rule 4.10. The questioning exercise cannot be bounded/terminated (*terminari*).[36]

However, he gives no explanation of this rule, or why it is introduced, and while he says that this rule is "just as in the preceding [types of] *obligationes*"[37], no similar rule is mentioned in his discussions of other types of *obligatio*.

[34] "*Ad omne antecedens ad dubitatum respondendum est 'falsum' vel 'proba' et nunquam 'verum'.*" [3, p. 224]

[35] "*Ad omne consequens ad dubitatum potest responderi 'verum est' vel 'proba' et nunquam 'falsum est'.*" [3, p. 224]

[36] "*Non possit terminari disciplinalis questio.*" [3, p. 226]

[37] "*Sicut in precedentibus obligationibus . . . .*" [3, p. 226]

We give an example of *dubitatio* in Figure 4, adapted from [3, pp. 223–224]. The Respondent has responded correctly in rounds (2) and (3), since (2) is identical with the *dubitatum* and (3) is equivalent to the *dubitatum*, and he also responds correctly in round (5) since black and white are exclusive, so Socrates's being white and his being black are repugnant (inconsistent) but he responds incorrectly in round (4) when he concedes that Socrates is black; for from the fact that it is doubtful whether Socrates is white it does not follow that Socrates is black, and further, by conceding that Socrates is black, the Respondent is later forced to deny the *dubitatum*, thus violating his primary obligation.

Nicholas's *dubitatio* has similar formal properties to *positio*. Provided that the *dubitatum* is neither a contradiction nor a tautology, it can be proved that the Respondent can *win* the disputation playing by Nicholas's rules for *dubitatio*: that is, there is never any case where he will be forced either to concede or to deny the *dubitatum* [34, Theorem 24].

4.4. *Impositio / Institutio / Appellatio*

The obligation involved in *impositio*, also called *institutio* or *appellatio*, functions in a relevantly different manner from the obligation in *positio*, *depositio*, or *dubitatio*. Whereas in these latter three, the Respondent's obligation involves how he is to respond to the *obligatum*, *impositio* involves an obligation to redefine certain terms or phrases. The anonymous author of *Obligationes Parisienses* notes that "*Institutio* is divided into certain *institutio* and uncertain or obscure *institutio*, for example if the name 'Marcus' is fixed that it might be a name of Socrates or Plato, but you would not know of which".[38] Lavenham defines *impositio* as an "obligation by means of which a term or proposition is assigned a [new] signification"[39], and gives the following examples:

> An example of the second: I impose that '*a*' signifies precisely that God exists. The whole "'*a*' signifies precisely that God exists" is what is obligated.[40]

[38] "*Institutio dividitur in institutionem certam et incertam sive occultam, ut ⟨si⟩ instituatur hoc nomen 'Marcus' quod sit nomen Sortis vel Platonis sed nescias utrius.*" [7, p. 28]

[39] "*. . . obligatio mediante qua terminus vel propositio imponitur ad significandum.*" [27, ¶21]

[40] "*Exemplum secundi: Impono quod 'a' praecise significet quod deus est. Hoc totum "'a' praecise significet quod deus est" est obligatum.*" [27, ¶2]

and

> [...] I impose that this term 'man' may be converted with this word 'donkey', or I impose that this proposition 'God exists' signifies precisely that man is donkey.[41]

In the first example, '*a*' is being instituted as the name of a proposition that signifies that God exists; likewise in the second example of the second quote, the phrase 'God exists' is instituted as the name of a proposition signifying that man is donkey; thus any time that 'God exists' is asserted in a disputation, it must be understood as meaning 'Man is donkey'. In the first example of the second quote, the institution is not at the level of propositions but at the level of words; it changes the meaning of the term 'man' so that it no longer means 'man' but instead means 'donkey'.

Impositio can take place in conjunction with any of *positio*, *depositio*, and *dubitatio*; that is, once a new imposition is introduced, then the Respondent may also further be obliged to concede, deny, or doubt the initial *obligatum* of the disputation. Lavenham gives two rules that govern *impositio* in the context of *positio* (the assumption being that adapting these rules to handle *impositio* in the context of *depositio* will be straightforward):

> And it should be known that in *positio* these rules must be observed: It is not because of the imposition of any proposition to what it has to signify that the response to them varied [...]. The second rule: Everything following from the *obligatum* by the imposition must be conceded, and everything repugnant denied.[42]

Simple *impositiones* like the ones given above are relatively easy; the only skill they require beyond the skills needed for *positio* is the skill to remember the new imposition of the term or proposition. However, much more complicated examples can be provided, such as the following (also due to Lavenham):

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[41] "*... impono quod iste terminus 'homo' convertatur cum ista dictione 'asinus', vel impono quod haec propositio 'Deus est' significet praecise quod homo est asinus.*" [27, ¶21]

[42] "*Et sciendum est quod in positione sunt istae regulae observandae: Non propter impositionem alicujus propositionis ad significandum est responsio ad illam varianda [...] Secunda regula: Omne sequens ex obligato ex impositione est concedendum et omne repugnans negandum.*" [27, ¶¶22–23]

> I impose that in every false proposition in which '*a*' is put down that it signifies only 'man' and that in every true proposition in which '*a*' is put down that it signifies only 'donkey', and that in every doubtful proposition in which '*a*' is put down that it signifies alternately with this disjunction 'man or non man'.[43]

Here, *a* is being used as a name for a term. This imposition may seem innocuous, but consider what happens with the proposition "Man is *a*" is put forward. Suppose the proposition is true. Then, it means "Man is donkey", which is impossible; hence, the proposition is false. But if it is false, it means "Man is man", which is in fact true! Thus, if it cannot be true or false, then it must be doubtful. But if it is doubtful, then it means 'Man is man or not man', which is true! No matter which assumption we make about the truth of the proposition, we are lead into contradiction.

4.5. *Petitio*

In *petitio*, the Opponent asks (petitions) the Respondent to respond in a certain way. *Petitio* is rarely treated at any length, because, as a number of authors (Nicholas of Paris, Marsilius of Inghen [23], Peter of Mantua [30], Paul of Venice) argue, *petitio* can be reduced to *positio*. In the mid-13th century, Nicholas of Paris says, when discussing how the verb *ponatur* 'to be put forward' initiates the obligation of the Respondent and the phrase *cedat tempus* 'the time is finished' disperses the obligation:

> Whence [...] the sense is, when the Opponent says '*ponatur*': I request that you are restricted to conceding the *positum* and everything convertible with it and following from it, etc.[44]

A century and a half later, Paul of Venice reduces *petitio* to *positio* in the following way:

> For when I say,
>
> I ask you to reply to *A*,
>
> this *petitio* makes the same claim as this *positio*:

[43] "*Impono quod in omni propositione falsa in qua ponitur 'a' quod significet hominem solum, et quod in omni propositione vera in qua ponitur 'a' quod significet asinum solum, et quod in omni propositione dubia in qua ponitur 'a' quod significet convertibiliter cum hoc disjuncto 'homo vel non homo'.*" [27, ¶24]

[44] "*Unde* [...] *sensus est, cum dicit opponens 'ponatur': peto ut restringaris ad concedendum positum et omne convertibile cum illo et consequens ad ipsum, etc.* [...]." [3, p. 183]

> I posit that you reply to *A*.
> Thus, in short, one should reply to all these species just as one would to a *positio*.[45] [25, p. 39]

Thus, from the disputational point of view, there is little more than cosmetic differences between *positio* and *petitio*.

However, for Burley, it is not *positio* but *institutio* (*impositio*) that *petitio* is connected to — *petitio* is the *actus* to *institutio*'s *habitus*, both of these governing what is noncomplex. He argues that "*petitio* is distinct from other species [of obligation], because a *petitio* posits the performance of an act that is mentioned in the statable thing [at issue], but the other species do not require this" [6, pp. 373–374].[46] An additional interesting fact about *petitio*, for Burley, is that, unlike *positio*, *petitio* can only engender an obligation for the Respondent with the Respondent's consent:

> So far it seems that *petitio* is not an obligation, for to require is not to obligate, because, unless the respondent consents, he is neither more nor less obligated in virtue of the opponent's requiring. We have to say [in reply] that *petitio* is an obligation — not just of any *petitio*, however, but only one that occurs with [the respondent's] consent.[47] [6, p. 374]

Further, because Opponent's initialization of the game is phrased as a request in *petitio*, there is a division into relative and absolute *petitiones* which is not present in *positio*. An example of an absolute *petitio* is the following: "I require you to grant that a man is a donkey"; an example of the second is "I require you to grant the first thing to be proposed by me". There is no way that this meta-level distinction could be made in *positio*.

4.6. *Rei veritas / sit verum*

This sixth type is rarely discussed by the medieval authors, and sometimes not even explicitly defined. As a result, it is difficult to give a

[45] "*Cum enim dico 'Peto te respondere ad A', adserit haec petitio sicut ista positio: 'Pono quod tu respondeas ad A'. Ita breviter quod ad omnes huiusmodi species respondendum est sicut ad positionem*" [25, p. 38]

[46] "*Dicendum quod petitio est distincta ab aliis speciebus, quia petitio ponit actum exerceri, qui ponitur in enuntiabili, sed hoc non petunt aliae species.*" [5, p. 41]

[47] "*Adhuc videtur quod petitio non sit obligatio, nam petere non est obligare, quia, nisi respondens consentiat propter petere opponentis, neque magis neque minus est obligatus. Dicendum quod petitio est obligatio, sed non quaelibet, sed solum petitio quae est cum consensum.*." [5, pp. 41–42]

precise explanation or characterization of this type. Instead we must look at particular examples given by various authors. For example, in Nicholas of Paris's *Obligationes*, the section on *sit verum* contains nothing more than a statement of *rei veritates* that cannot be sustained.[48] One explanation for this fact is that, for authors following Burley, if *sit verum* "obligates [the Respondent] to an act [as opposed to a disposition] and [...] covers what is complex",[49] then the only way it differs from *positio* is in this distinction between an obligation to a disposition and an obligation to an act — a distinction which is never clearly spelled out — and hence the actual protocol for the disputation, once the initial obligation is filled out, will not differ between the two.

Most discussions of *sit verum* focus on epistemic aspects of the disputation [3, p. 166], [32, p. 320]. For instance, Paul of Venice gives the following example of *sit verum*: "Let it be true that you know that you are replying" [25, p. 45].[50] More illustrative is the first example of a *rei veritas* that cannot be sustained that Nicholas gives:

> Whence it is customary to say that this cannot be sustained: 'the truth of things is that only Socrates knows that the king is in Paris'. For if it were sustained, then a contradiction follows. For if you know that only Socrates knows that the king is in Paris, you know that Socrates knows nothing except the truth; therefore you know that the king is in Paris is true, and thus you know that the king is in France, therefore not only Socrates knows this.[51]

This example shows an interesting resemblance to Fitch's and Moore's paradoxes [11, 24], namely that you cannot (truthfully) tell someone "φ, and you don't know (or believe) that φ", for once you tell them that

[48] "*Videntur autem multe rei veritates esse que sustinende non sunt.*" [3, p. 233]

[49] This is Burley's definition; it is very similar to how Paul of Venice defines it: "Every *obligatio* obligates a person either to an act or to a disposition. If it obligates a person to an act, this can happen in two ways. Either it obligates a person to a simple act, and this is a *petitio*, or to a complex act, and this is a *sit verum*" [25, p. 45]. (*Omnis obligatio vel obligat ad actum vel ad habitum. Si ad actum: hoc dupliciter: vel ad actum incomplexum, sic est petitio; vel ad actum complexum, sic est sit verum* [25, p. 44]).

[50] "*Sit verum te scire te respondere.*" [25, p. 44]

[51] "*Unde solet dici quod hec non est sustinenda: 'rei veritas est quod solus Sortes scit regem esse Parisius'. Si enim sustineatur, sequitur contradictio. Si enim scis quod solus Sortes scit regem esse Parisius, scis quod Sortes scit nisi verum; ergo scis quod regem esse Parisius est verum, et ita scis regem esse Parisius, ergo non solus Sortes scit.*" [3, p. 233]

φ is true, it is no longer the case that they do not know or believe φ (assuming basic constraints on how knowledge and beliefs are formed).

However, one text offers a tantalizingly interesting distinction between *rei veritatis* and *positio*. The anonymous author of the *Obligationes Parisienses* says:

> And *rei veritatis* differs from *positio* because when *rei veritatis* has been done, then concerning any thing irrelevant or not following, it is not refused, but on the other hand if *positio* has been done, then it is to be refused. When it is said '*rei veritatis* is that the Antichrist exists', then concerning this: 'The Antichrist is white' the response is 'prove it!', but if *positio* has been done the response to the same is: 'It is false!' ".[52]

This shows a marked similarity to the counterfactual approach of Kilvington. What the author is pointing out is that when a disputation is prefaced with a statement about "the truth of things" (*rei veritas*), by saying "let it be true that [...]" (*sit verum*), this effectively changes the truth value of the proposition during the disputation — in a way that conceding or denying a proposition does not. Suppose that, in *positio*, Respondent has conceded 'Antichrist exists' (even though this is false). If, then, Opponent puts forward 'Antichrist is white', this statement is irrelevant, and because it is false (since Antichrist does not exist, he cannot be colored), it should be denied. However, if "the truth of things" is that Antichrist exists, then when Opponent asserts that he is white, the correct answer is "prove it". If Antichrist exists, then he must be colored, but whether he is colored white or colored black is doubtful.

5. Cooperative and uncooperative games

Above we noted that the Aristotelian dialogues can be divided into cooperative and uncooperative (or strategic) games, and that at least one medieval author considered obligational disputations to straddle this division. How else do *obligationes* differ from Aristotelian dialogue games, and what significance do these differences have for the interactive nature of logic?

[52] "*Et* rei veritas *differt a positione quia, rei veritate facta circa aliquod impertinens, non sequens, non est negandum, facta autem positione est negandum. Unde dicto* 'rei veritas est quod Antichristus est', *ad hanc* 'Antichristus est albus' *respondendum est* 'proba!', *sed facta positione ad eandem respondendum est:* 'falsum est!' ." [7, p. 28]

First, while it is clear that *obligationes* fall in the "question-answer" type of disputation, even if it is clear that they are neither purely didactic nor purely contentious, they differ in an important respect. In Aristotelian dialogue games, the thesis being disputed is generally going to be *true* or at least *believed to be true*. But, this is not the case with *obligationes*; as a number of authors point out, the only *positiones* that are of interest are those where the initial statement put forward is *false*; a *positio* with a true *positum* will be an uninteresting exercise. It is for this reason that some modern scholars have argued that *obligationes* represent a type of counterfactual or thought-experimental reasoning [19, 28]. However, such analyses fail to provide an explanation for the interactive nature of the *obligationes*; were they really intended to provide a method of counterfactual reasoning or of constructing and evaluating thought experiments, there would be no need for them to be two-person dialogue games, since both of these types of reasoning can be done by a single agent in isolation.[53] A more felicitous explanation of the utility of obligational reasoning is provided by King, where he explains what he sees as the apparent "content-freeness" of obligational disputations by pointing out that "they operate at a higher level of logical generality than that at which substantive debate occurs. If this is correct, then actual obligational moves — perhaps even recognized as such — are the vehicle whereby real argument takes place" [20, p. 6], and thus *obligationes* provide a "meta-methodology" for reasoning [20, p. 7]. In recent work we have shown how this idea of a meta-methodology can be made formal and precise, by explaining how rules for different types of *obligationes* can be viewed as giving rise to protocols governing systems of formal dialogues [36].

Second, as we stressed in §2, even when the rules are changed, the focus in all of these games is the maintenance of (some level of) consistency. Obligational disputations thus differ in a crucial way from modern game or dialogue approaches to logic (à la Lorenzen [22] and Hintikka [13]) in that they are not intended to give semantic meaning to the logical connectives, or to demonstrate the validity of a proposition. Instead, the rules of inference for reasoning must be known in advance by both the Opponent and the Respondent, and an obligational disputation about

[53] Yrjönsuuri also believes that "medieval authors did not generally accept the idea of employing counterfactual reasoning in the rules of obligations" [40, p. 72], and he says this is one reason why Kilvington's theory, with its markedly counterfactual language for evaluating irrelevant propositions, was not widely accepted.

a sentence φ can be understood as testing the Respondent's ability to reason about φ, either propositionally (i.e., about the truth or falsity of φ) or at the meta-level (e.g., about whether φ is known or in doubt to him, or whether he is obliged to concede, deny, or doubt φ). Thus, one of the obligations of the Respondent can be understood as building a consistent world-description, starting from (in *positio*), a false but not logically impossible proposition. In many 13th century treatises, many authors note that, if the *positum* is in fact false during the time of the disputation, and if all answers during the disputation must be directed to the same instant, then that instant to which all answers are directed cannot be the present instant. Thus, *positiones* according to these rules can be seen as developing a consistent description of some instant of time other than the present instant. (Note that, contra Yrjönsuuri [40, p. 71], this instant need not be some "imagined future instant"; it could be any instant, including ones not temporally connected to the present instant, so long as it is not the present instant.)

But the obligation of the Respondent is twofold; it is not sufficient just to maintain a set of answers consistent with the *positum*, but he must also adhere to the rules. Thus, there is a type of dialectical obligation that the Respondent also has; for he could maintain consistency, in any type of disputation, merely by responding "I doubt it" to every proposition put forward after the *positum*, but in doing so he is failing his more general obligation to follow the rules, which require him to concede and deny some propositions. In fact, we can see that the obligation of maintaining consistency depends on the obligation to follow the rules. These two obligations are why there are two different ways that the Respondent can make mistakes: He can make a mistake by applying a rule incorrectly, or he can make a mistake by taking up an inconsistent *obligatum*. A similar distinction is made in Aristotle: "for one may, no doubt, distinguish between the mistake of taking up a wrong thesis to start with, and that of not maintaining it properly, when once taken up" [2, p. 268], but while the latter mistake is considered a worse mistake in Aristotelian dialogues, it is the former mistake that garnishes the most censure from *obligationes* authors [40, p. 65]. This is why most authors focus so much on giving examples where a seemingly innocuous *obligatum* in fact contains a hidden contradiction — such as the second example of *positio* from Burley and the third example of *institutio* from Lavenham that we saw above.

Fourth, while in an Aristotelian dialectical disputation, "the answerer should say 'Yes' if the statement put by the questioner is either accepted

by everyone, or by the majority, by the wise, by the practical experts, or by himself and say 'No' otherwise" [16, p. 245] (see [2, p. 268]), there is no such provision for appeal to authority in an *obligatio*. Instead, the emphasis always is — and this is perhaps the one aspect in which *all* treatises on *obligationes* agree — on the inferential relations between propositions and the dialectical commitments [12] that the player's moves engender. There is one exception to this: a species of *positio* which is very little discussed called *positio vicaria*, in which the Respondent is obliged "to answer according to the viewpoint of some other person"; in Burley, Sherwood, and Nicholas of Paris, a single example is given, namely, according to the views of Zeno, with respect to the proposition that nothing movies [3, p. 166]. (In Nicholas, this obligation is a type of *petitio*, since the Respondent is being asked to respond in a certain way.) But this was clearly an unimportant variant, meriting little discussion and consideration.

Fifth, Aristotle gives a rule for the answerer concerning when he is allowed to break off the game: "The answerer should break off the game if the questioner repeats the same question or fails to reason" [2, Book VIII, ch. 5, 158a29–30]. Such a rule could be adopted in *responsio nova*-style *obligationes*, which have a static definition of pertinence, for it can be proven that if the Opponent repeats a proposition in such an *obligatio*, the Respondent will always give the same answer. Thus, if the Opponent has reached a point in his attempts to trick the Respondent into conceding a contradiction where he must repeat himself, we can say that Respondent has, in a sense, won. But with the dynamic definition of pertinence found in many of the *responsio antiqua* authors, it is reasonable for Opponent to repeat his propositions, given that Respondent may be forced to give different answers to the propositions at different times. However, in *positio* or *depositio*, once Respondent has (correctly) conceded or denied a proposition, then Opponent would gain nothing by repeating the proposition, for Respondent's answer to it cannot change.

6. Conclusion

We have seen a number of different examples of "interactive logic" in the Middle Ages, all species of the disputation game *obligatio*. Theories of *obligationes* were developed from the beginning of the 13th century, and continued to be an active field of research through the beginning of the 15th century. While rooted in Aristotelian theories of debate

and dialogue, we have shown how *obligationes* differ in many important respects. They represent one of the keynote contributions of logicians in the Middle Ages, displaying a recognition of the importance of interaction in logical contexts and the way that interactive logic differs from single-agent inference. Thus, the obligational disputations share characteristics with both cooperative and competitive games.

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