TABLE 2.2 Fourier Series Coefficients for Selected Waveforms

Periodic Waveform	Symmetry	F_n [†]
1. *Symmetric square wave* $\begin{cases} +1 & \lvert t\rvert < T/4 \\ -1 & T/4 \le \lvert t\rvert < T/2 \end{cases}$	*even*	$\begin{cases} \text{Sa}(n\pi/2) & n \neq 0 \\ 0 & n = 0 \end{cases}$
2. *Rectangular pulse train* $\begin{cases} +1 & \lvert t\rvert < \tau/2 \\ 0 & \tau/2 \le \lvert t\rvert < T/2 \end{cases}$	*even*	$\frac{\tau}{T}\,\text{Sa}(n\pi\tau/T)$
3. *Symmetric triangular wave* $1 - 4\lvert t\rvert/T, \quad \lvert t\rvert < T/2$	*even*	$\begin{cases} \text{Sa}^2(n\pi/2) & n \neq 0 \\ 0 & n = 0 \end{cases}$
4. *Symmetric sawtooth wave* $2t/T, \quad \lvert t\rvert < T/2$	*odd*	$\begin{cases} j(-1)^n/(n\pi) & n \neq 0 \\ 0 & n = 0 \end{cases}$
5. *Half-wave rectified sinusoid* $\begin{cases} \sin \omega_0 t & 0 \le t < T/2 \\ 0 & -T/2 \le t < 0 \end{cases}$		$\begin{cases} \dfrac{1}{\pi(1-n^2)} & n \text{ even} \\ -j/4 & n = \pm 1 \\ 0 & \text{otherwise} \end{cases}$
6. *Full-wave rectified sinusoid* $\lvert\sin \omega_0 t\rvert$	*even*	$\begin{cases} \dfrac{2}{\pi(1-n^2)} & n \text{ even} \\ 0 & \text{otherwise} \end{cases}$

[†] $\text{Sa}(x) = (\sin x)/x$.

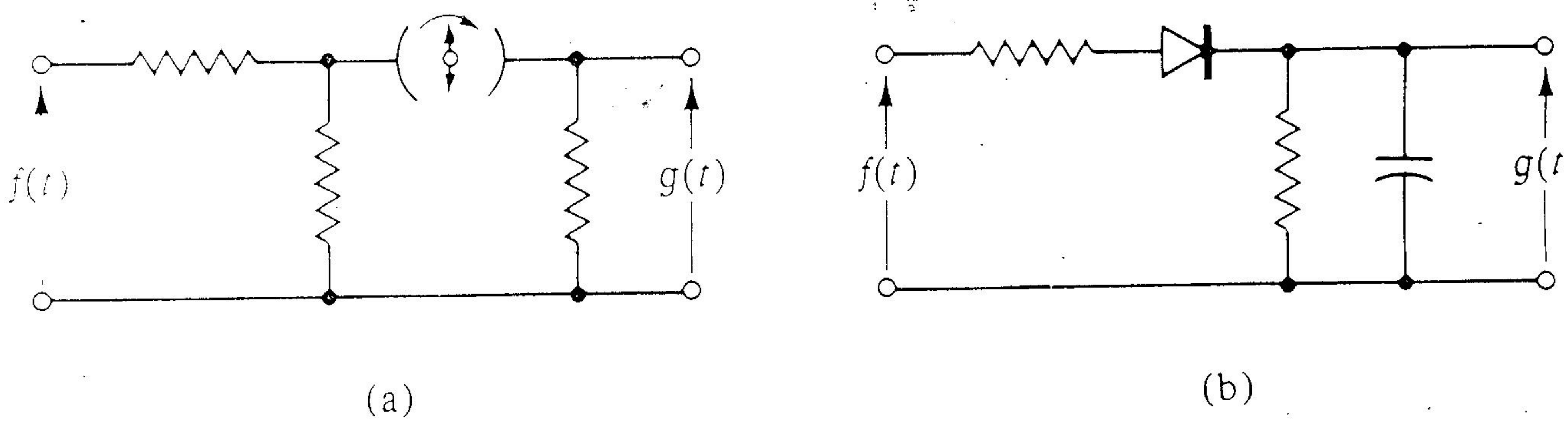

Figure 2.3 Examples of systems: (a) Linear, time-varying; (b) nonlinea time-invariant.

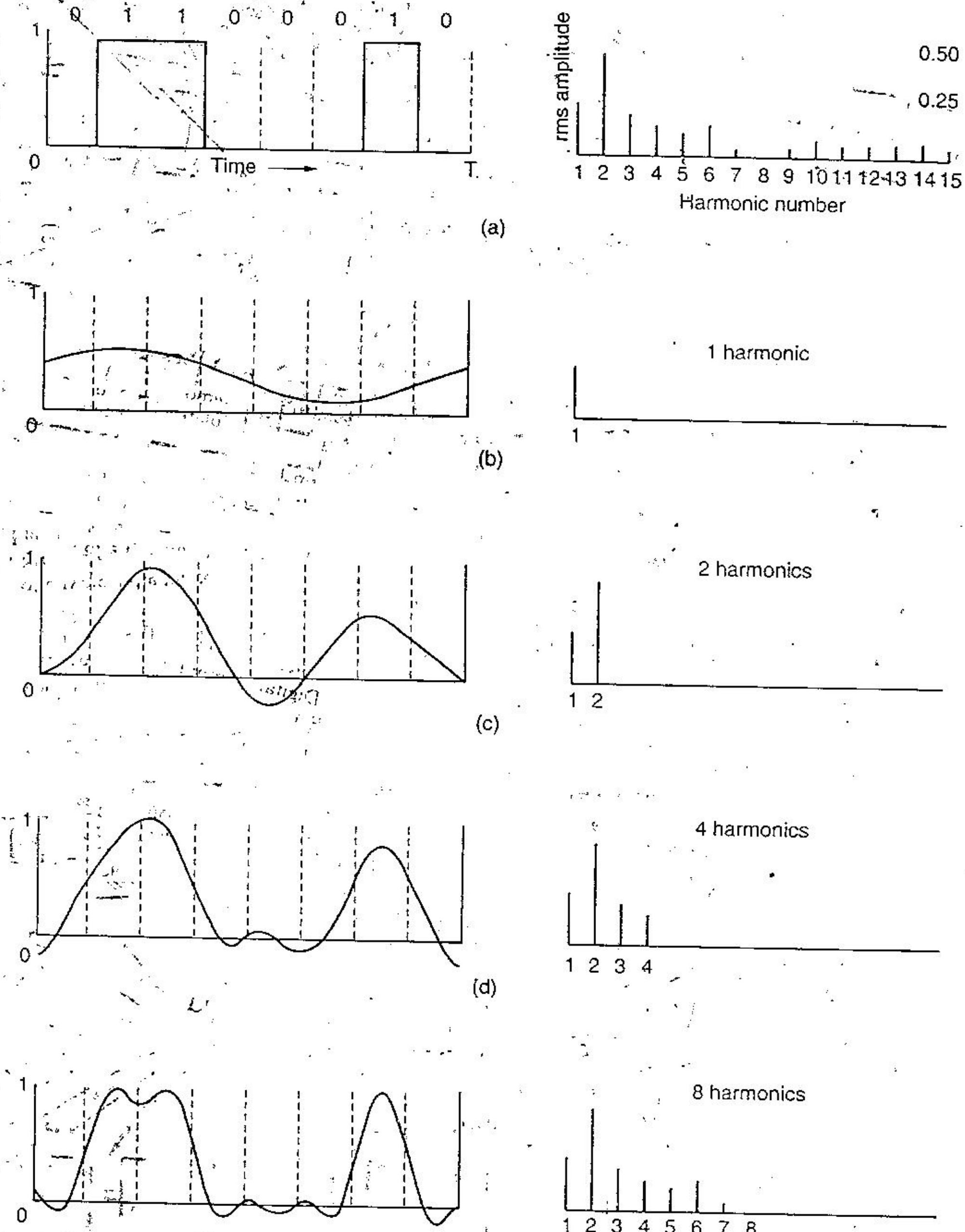

Fig. 2-1. (a) A binary signal and its root-mean-square Fourier amplitudes. (b)-(e) Successive approximations to the original signal.

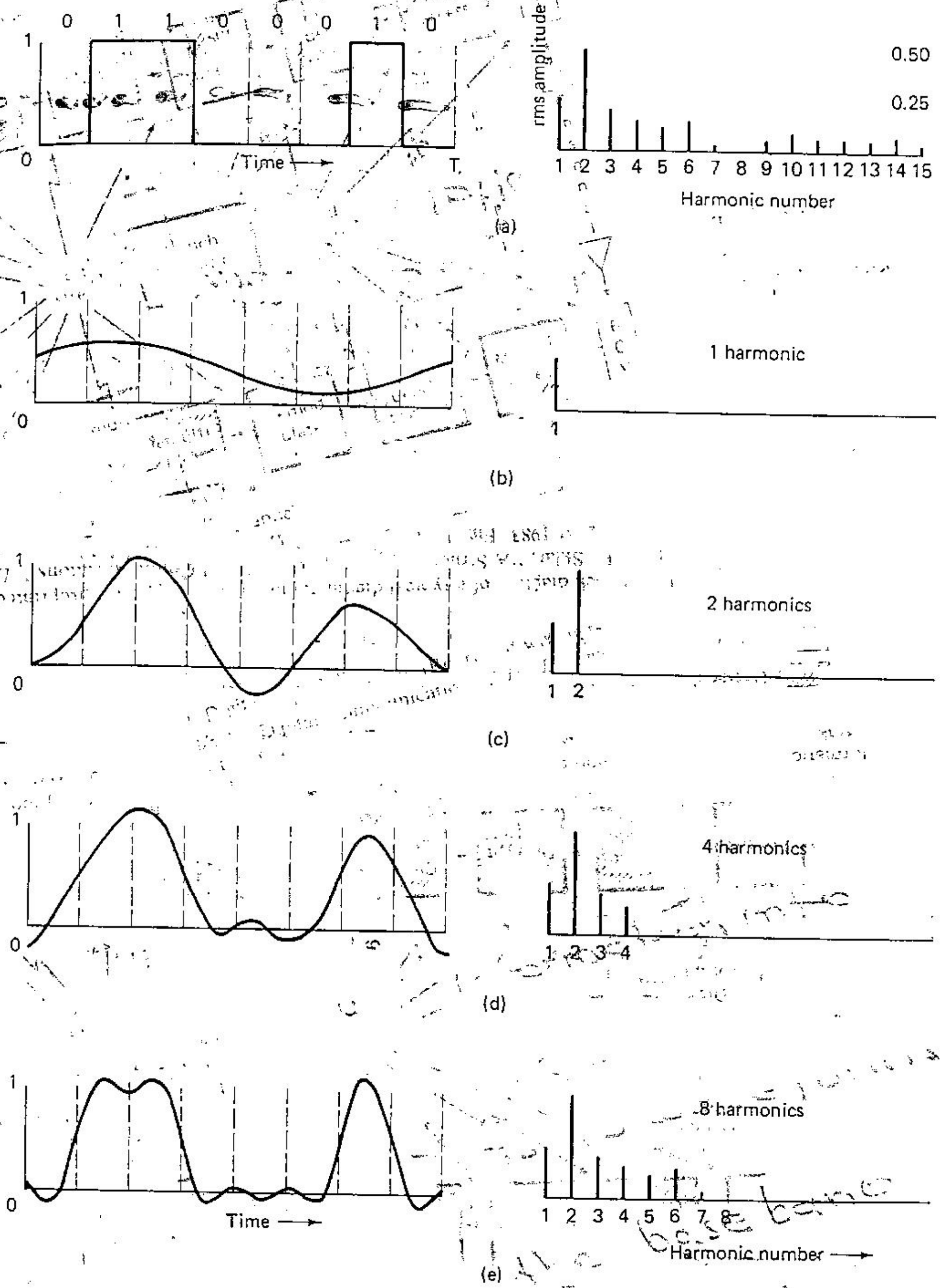

Fig. 2-1. (a) A binary signal and its rms Fourier amplitudes. (b)-(e) Successive approximations to the original signal.

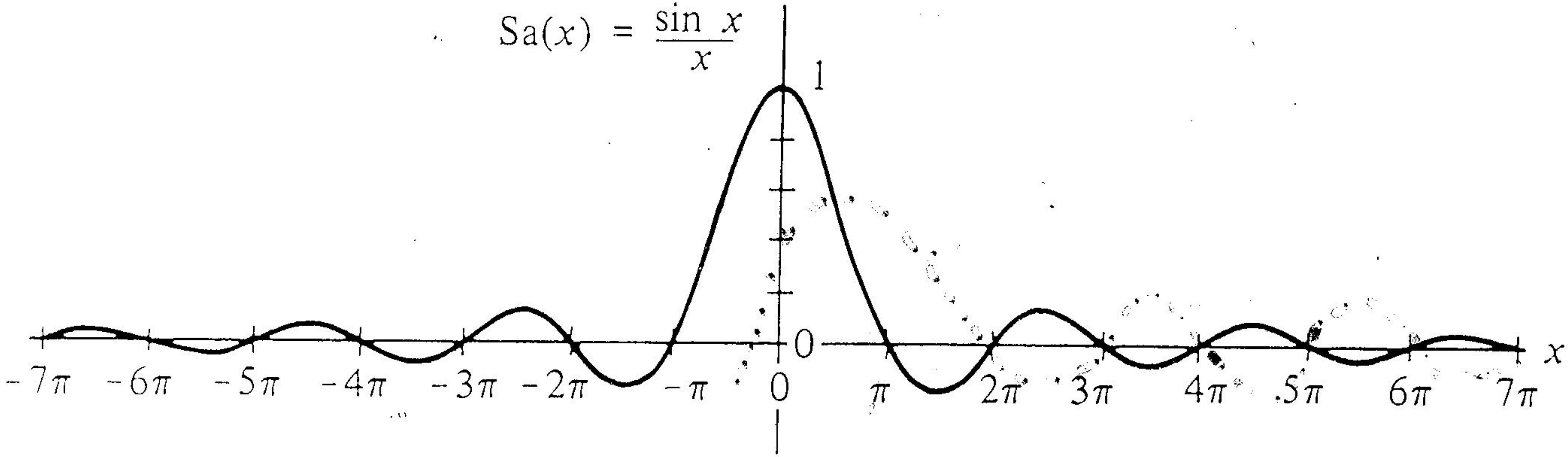

Figure 2.22 The function Sa(x).

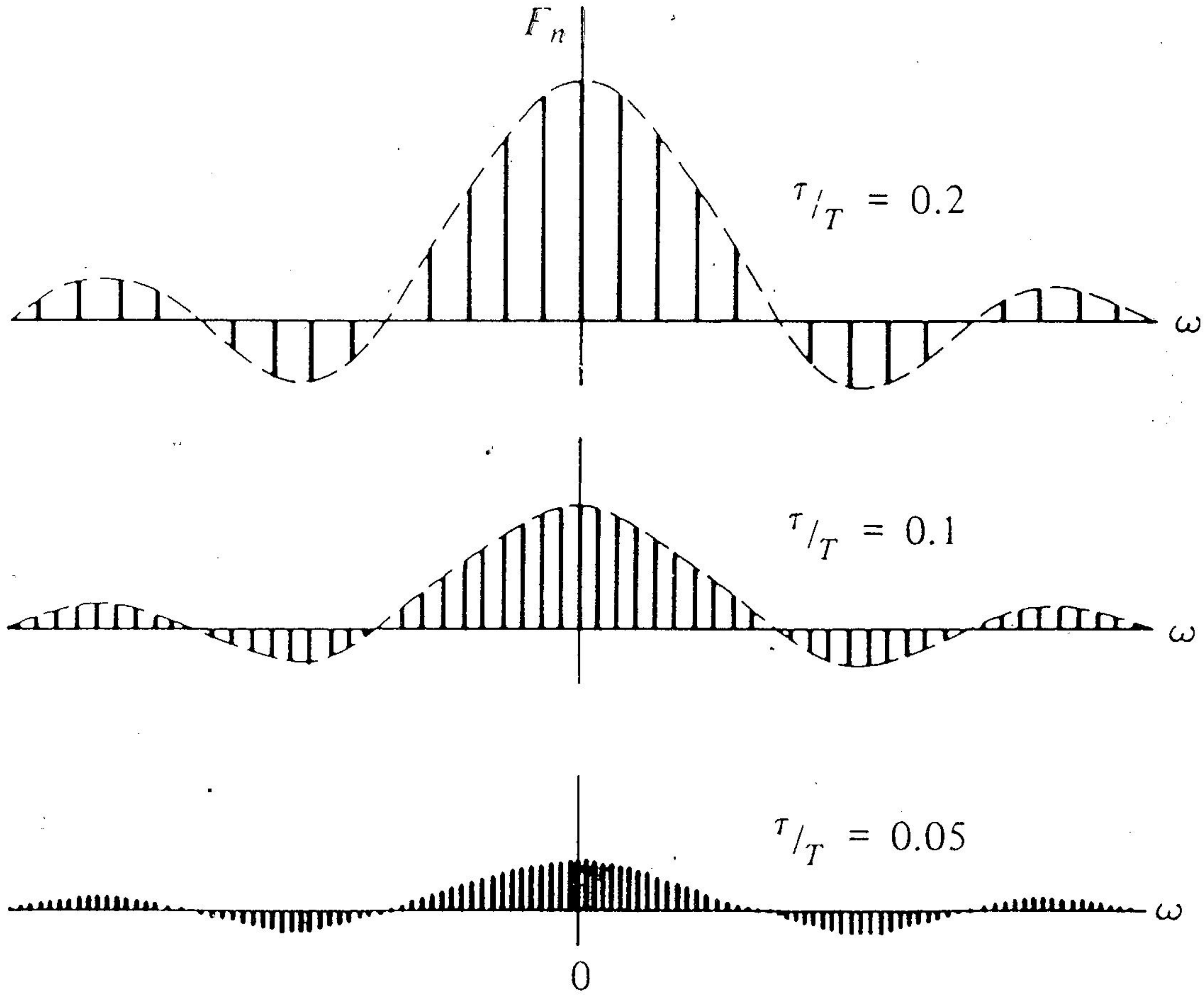

Figure 2.23 Amplitude spectra for various values of τ/T, τ fixed.

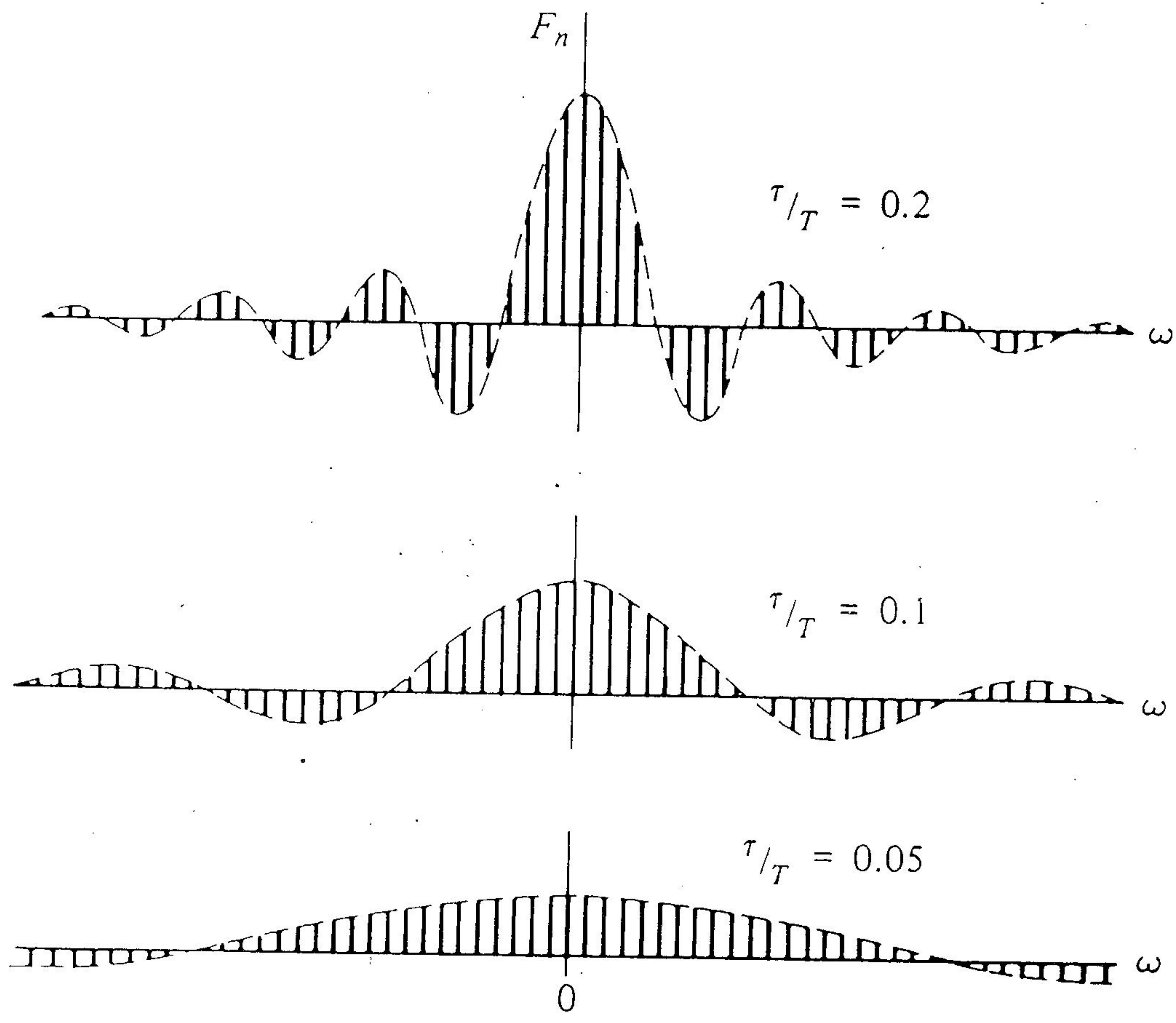

Figure 2.24 Amplitude spectra for various values of τ/T, T fixed.

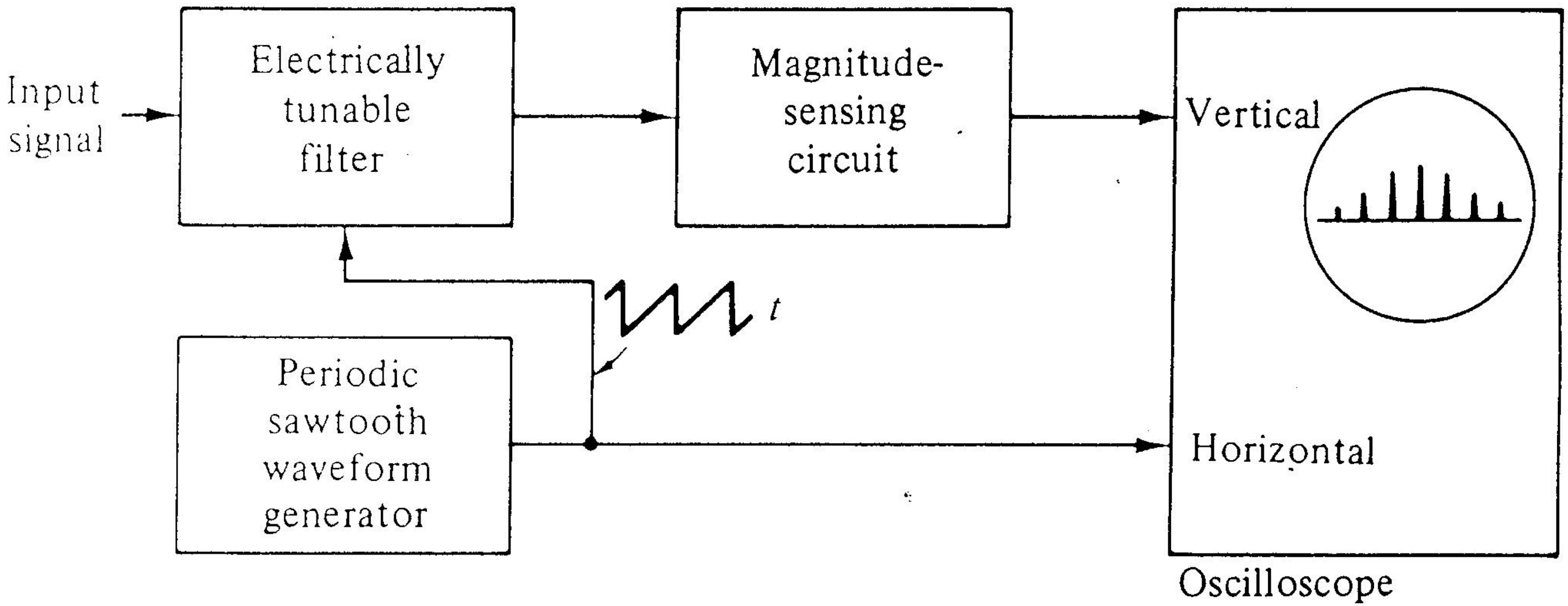

Figure 2.25 Block diagram of a scanning spectrum analyzer.

Selected Fourier Transform Pairs

	$f(t)$	$F(\omega) = \mathscr{F}\{f(t)\}$		$f(t)$	$F(\omega) = \mathscr{F}\{f(t)\}$
1.	$e^{-at}u(t)$	$1/(a + j\omega)$	11.	$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
2.	$te^{-at}u(t)$	$1/(a + j\omega)^2$	12.	$\sin \omega_0 t$	$-j\pi[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
3.	$e^{-a\|t\|}$	$2a/(a^2 + \omega^2)$	13.	$\text{rect}(t/\tau)$	$\tau\, \text{Sa}(\omega\tau/2)$
4.	$e^{-t^2/(2\sigma^2)}$	$\sigma\sqrt{2\pi}\, e^{-\sigma^2\omega^2/2}$	14.	$\frac{W}{2\pi}\, \text{Sa}(Wt/2)$	$\text{rect}(\omega/W)$
5.	$\text{sgn}(t)$	$2/(j\omega)$	15.	$\frac{W}{\pi}\, \text{Sa}(Wt)$	$\text{rect}(\omega/2W)$
6.	$j/(\pi t)$	$\text{sgn}(\omega)$	16.	$\Lambda(t/\tau)$	$\tau[\text{Sa}(\omega\tau/2)]^2$
7.	$u(t)$	$\pi\delta(\omega) + 1/(j\omega)$	17.	$\frac{W}{2\pi}[\text{Sa}(Wt/2)]^2$	$\Lambda(\omega/W)$
8.	$\delta(t)$	1	18.	$\cos(\pi t/\tau)\, \text{rect}(t/\tau)$	$\frac{2\tau}{\pi} \frac{\cos(\omega\tau/2)}{1 - (\omega\tau/\pi)^2}$
9.	1	$2\pi\delta(\omega)$	19.	$\frac{2W}{\pi^2} \frac{\cos(Wt)}{1 - (2Wt/\pi)^2}$	$\cos[\pi\omega/(2W)]\, \text{rect}[\omega/(2W)]$
10.	$e^{\pm j\omega_0 t}$	$2\pi\delta(\omega \mp \omega_0)$	20.	$\delta_T(t)$	$\omega_0\delta_{\omega_0}(\omega)$, where $\omega_0 = 2\pi/T$

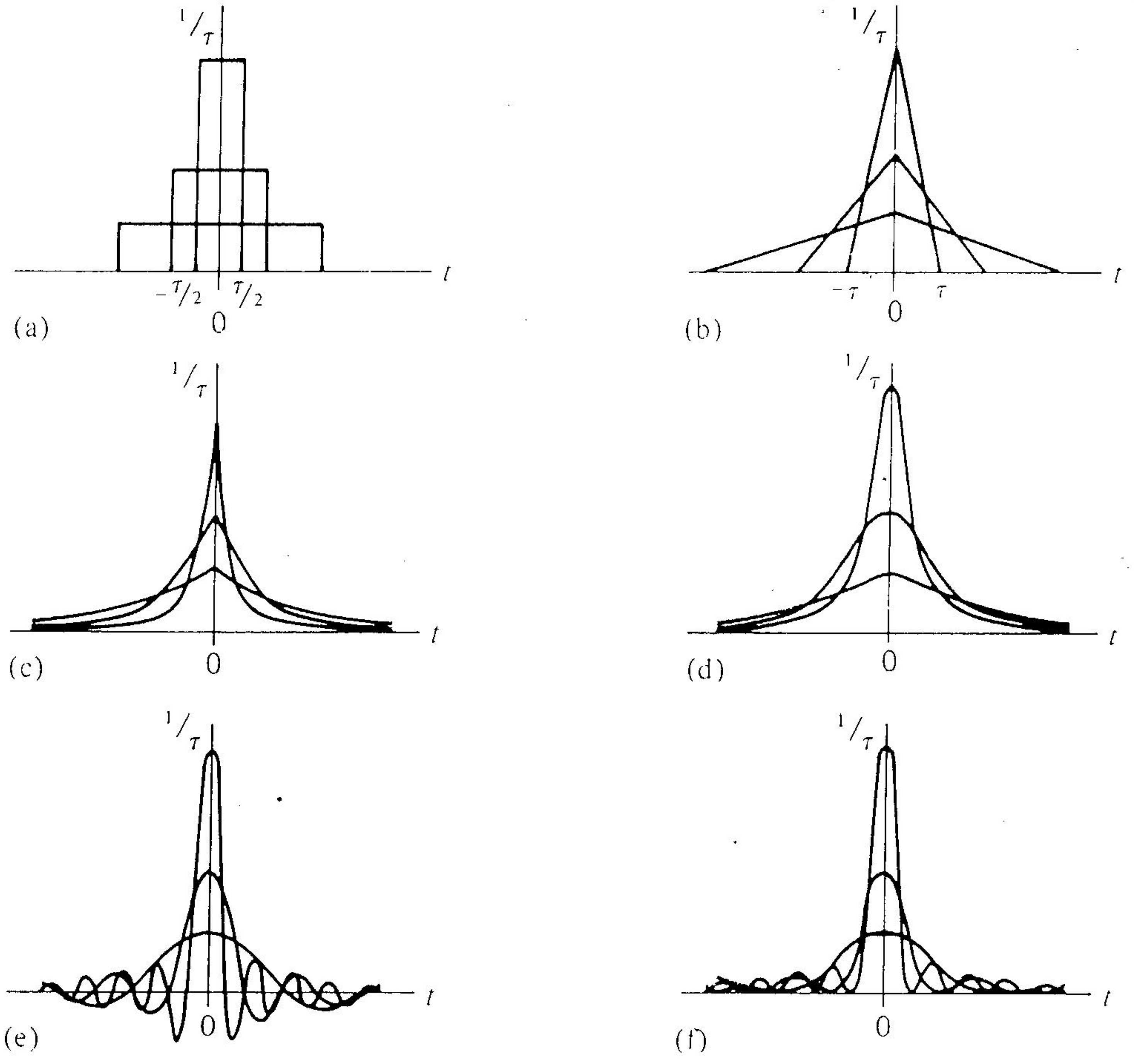

Figure 2.30 Function sequence definition of the impulse function: (a) rectangular pulse; (b) triangular pulse; (c) two-sided exponential;

Some Fourier Transforms Corresponding to Given Mathematical Operations

Operation	$f(t)$	$\leftrightarrow$	$F(\omega)$
Linearity (superposition)	$a_1 f_1(t) + a_2 f_2(t)$		$a_1 F_1(\omega) + a_2 F_2(\omega)$
Complex conjugate	$f^*(t)$		$F^*(-\omega)$
Scaling	$f(\alpha t)$		$\frac{1}{\lvert\alpha\rvert} F\left(\frac{\omega}{\alpha}\right)$
Delay	$f(t - t_0)$		$e^{-j\omega t_0} F(\omega)$
Frequency translation	$e^{j\omega_0 t} f(t)$		$F(\omega - \omega_0)$
Amplitude modulation	$f(t) \cos \omega_0 t$		$\frac{1}{2}F(\omega + \omega_0) + \frac{1}{2}F(\omega - \omega_0)$
Time convolution	$\int_{-\infty}^{\infty} f_1(\tau) f_2(t - \tau)\, d\tau$		$F_1(\omega) F_2(\omega)$
Frequency convolution	$f_1(t) f_2(t)$		$\frac{1}{2\pi} \int_{-\infty}^{\infty} F_1(u) F_2(\omega - u)\, du$
Duality: time-frequency	$F(t)$		$2\pi f(-\omega)$
Symmetry: even-odd	$f_e(t)$		$F_e(\omega)$ [real]
	$f_o(t)$		$F_o(\omega)$ [imaginary]
Time differentiation	$\frac{d}{dt} f(t)$		$j\omega F(\omega)$
Time integration	$\int_{-\infty}^{t} f(\tau)\, d\tau$		$\frac{1}{j\omega} F(\omega) + \pi F(0)\, \delta(\omega)$, where $F(0) = \int_{-\infty}^{\infty} f(t)\, dt$

$$\lim_{T\to\infty} f_T(t) = f(t) \,. \tag{3.1}$$

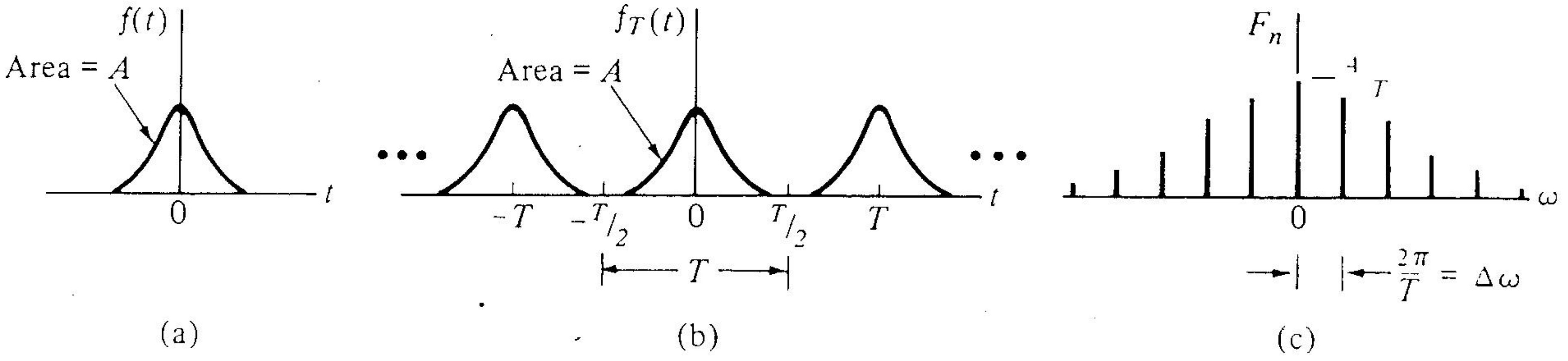

Figure 3.1 Generation of a periodic function and its line spectrum.

TABLE 3.1 Some Selected Fourier Transform Pairs

	$f(t)$	$F(\omega) = \mathcal{F}\{f(t)\}$
1.	$e^{-at}u(t)$	$1/(a + j\omega)$
2.	$te^{-at}u(t)$	$1/(a + j\omega)^2$
3.	$e^{-a\|t\|}$	$2a/(a^2 + \omega^2)$
4.	$e^{-t^2/(2\sigma^2)}$	$\sigma\sqrt{2\pi}\, e^{-\sigma^2\omega^2/2}$
5.	$\operatorname{sgn}(t)$	$2/(j\omega)$
6.	$j/(\pi t)$	$\operatorname{sgn}(\omega)$
7.	$u(t)$	$\pi\delta(\omega) + 1/(j\omega)$
8.	$\delta(t)$	1
9.	1	$2\pi\delta(\omega)$
10.	$e^{\pm j\omega_0 t}$	$2\pi\delta(\omega \mp \omega_0)$
11.	$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
12.	$\sin \omega_0 t$	$-j\pi[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
13.	$\operatorname{rect}(t/\tau)$	$\tau \operatorname{Sa}(\omega\tau/2)$
14.	$\frac{W}{2\pi}\operatorname{Sa}(Wt/2)$	$\operatorname{rect}(\omega/W)$
15.	$\frac{W}{\pi}\operatorname{Sa}(Wt)$	$\operatorname{rect}(\omega/(2W))$
16.	$\Lambda(t/\tau)$	$\tau[\operatorname{Sa}(\omega\tau/2)]^2$
17.	$\frac{W}{2\pi}[\operatorname{Sa}(Wt/2)]^2$	$\Lambda(\omega/W)$
18.	$\cos(\pi t/\tau)\operatorname{rect}(t/\tau)$	$\frac{2\tau}{\pi}\frac{\cos(\omega\tau/2)}{1 - (\omega\tau/\pi)^2}$
19.	$\frac{2W}{\pi^2}\frac{\cos(Wt)}{1 - (2Wt/\pi)^2}$	$\cos[\pi\omega/(2W)]\operatorname{rect}[\omega/(2W)]$
20.	$\delta_T(t)$	$\omega_0\,\delta_{\omega_0}(\omega)$, where $\omega_0 = 2\pi/T$

TABLE 3.2 Some Fourier Transforms Corresponding to Given Mathematical Operations

Operation	$f(t)$ $\leftrightarrow$	$F(\omega)$
Linearity (superposition)	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(\omega) + a_2 F_2(\omega)$
Complex conjugate	$f^*(t)$	$F^*(-\omega)$
Scaling	$f(\alpha t)$	$\frac{1}{\lvert\alpha\rvert} F\left(\frac{\omega}{\alpha}\right)$
Delay	$f(t - t_0)$	$e^{-j\omega t_0} F(\omega)$
Frequency translation	$e^{j\omega_0 t} f(t)$	$F(\omega - \omega_0)$
Amplitude modulation	$f(t) \cos \omega_0 t$	$\frac{1}{2}F(\omega + \omega_0) + \frac{1}{2}F(\omega - \omega_0)$
Time convolution	$\int_{-\infty}^{\infty} f_1(\tau) f_2(t - \tau)\, d\tau$	$F_1(\omega) F_2(\omega)$
Frequency convolution	$f_1(t) f_2(t)$	$\frac{1}{2\pi} \int_{-\infty}^{\infty} F_1(u) F_2(\omega - u)\, du$
Duality: time-frequency	$F(t)$	$2\pi f(-\omega)$
Symmetry: even-odd	$f_e(t)$	$F_e(\omega)$ [real]
	$f_o(t)$	$F_o(\omega)$ [imaginary]
Time differentiation	$\frac{d}{dt} f(t)$	$j\omega F(\omega)$
Time integration	$\int_{-\infty}^{t} f(\tau)\, d\tau$	$\frac{1}{j\omega} F(\omega) + \pi F(0)\, \delta(\omega)$, where $F(0) = \int_{-\infty}^{\infty} f(t)\, dt$

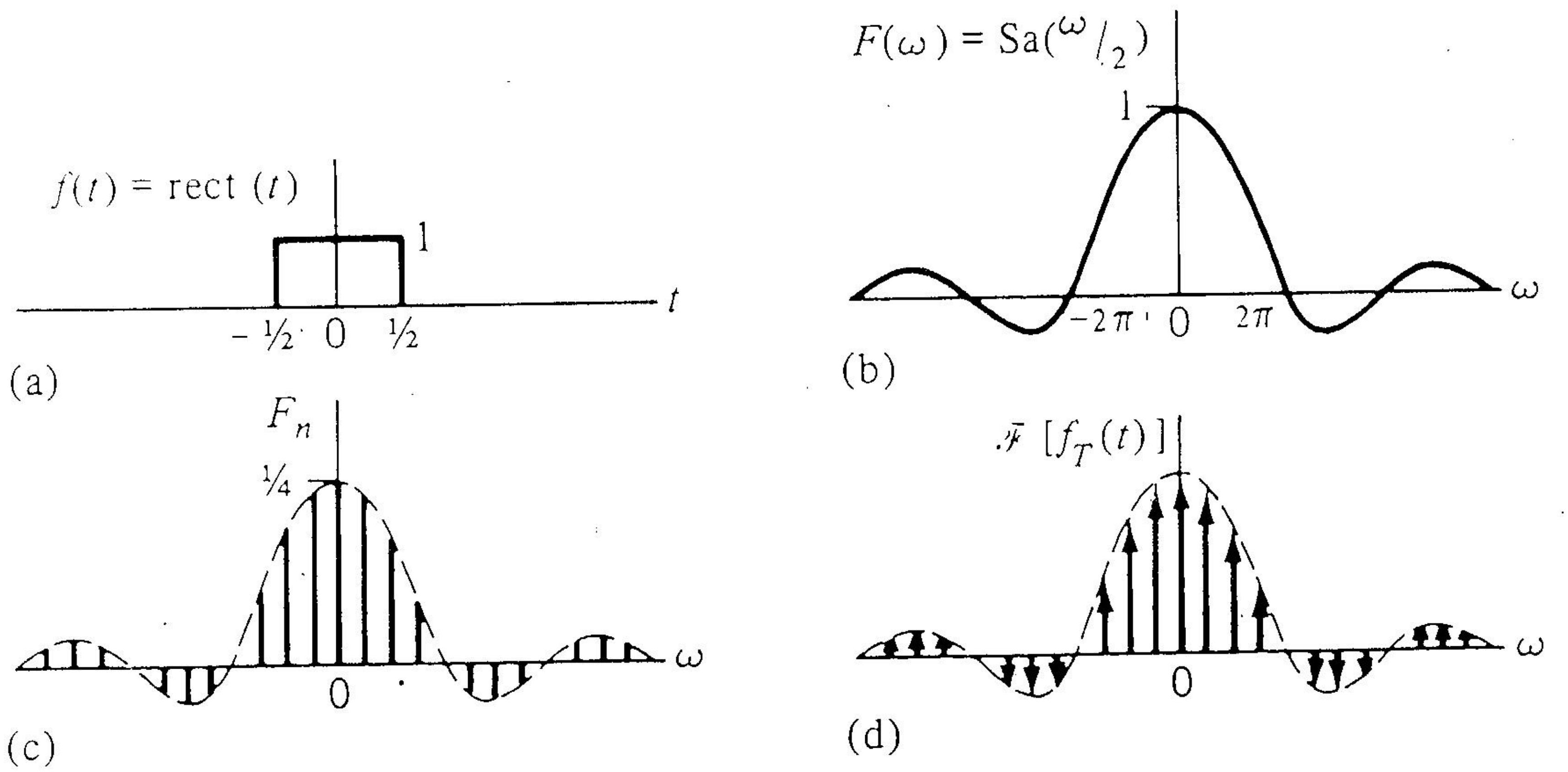

Figure 3.2 (a) The unit gate function and (b) its Fourier transform. (c) The line spectrum and (d) spectral density of the periodic gate function.

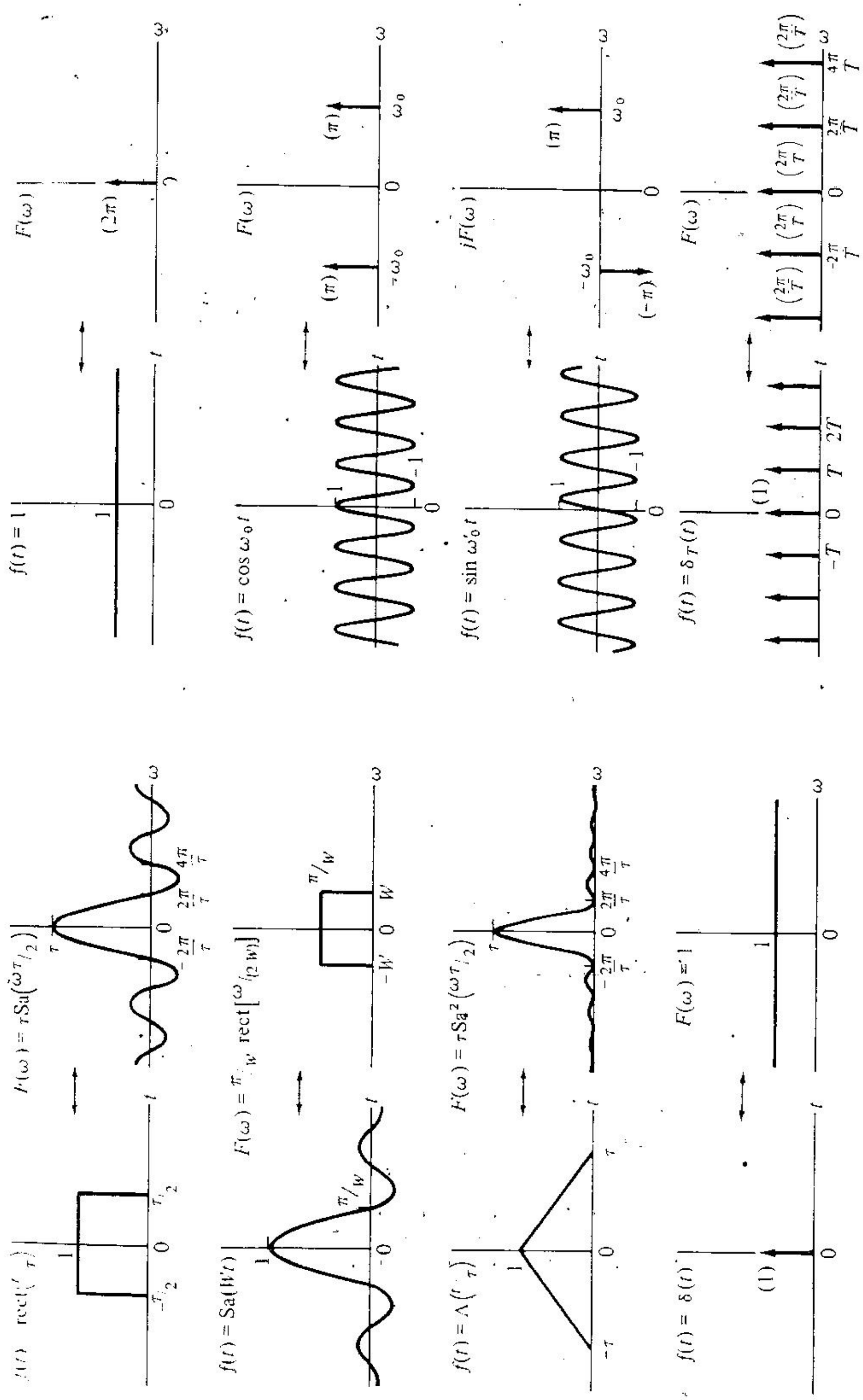

Figure 3.3 Some functions of time and their spectral-density functions.

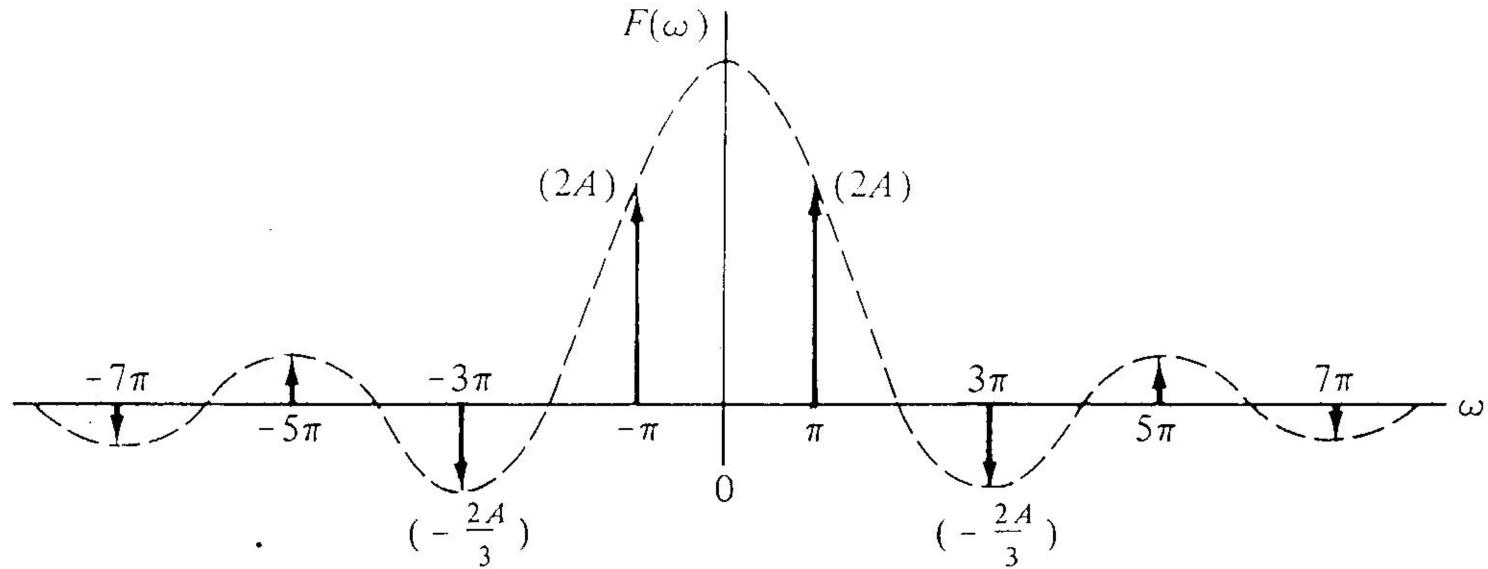

Figure 3.4 The spectral-density function of a periodic square wave with zero average value.

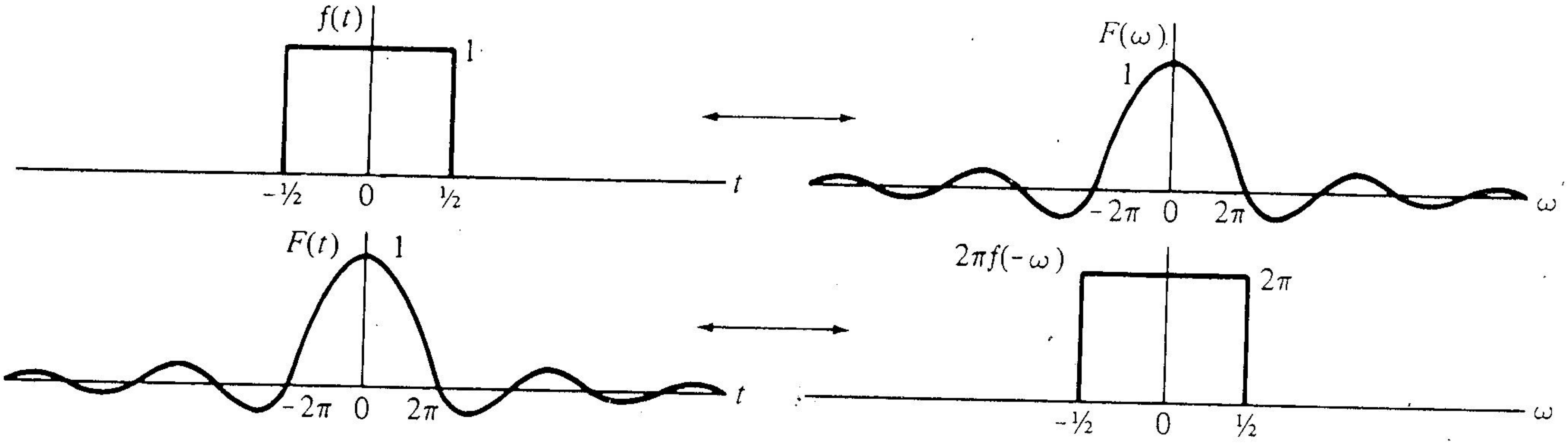

Figure 3.5 Duality of the Fourier transformation.

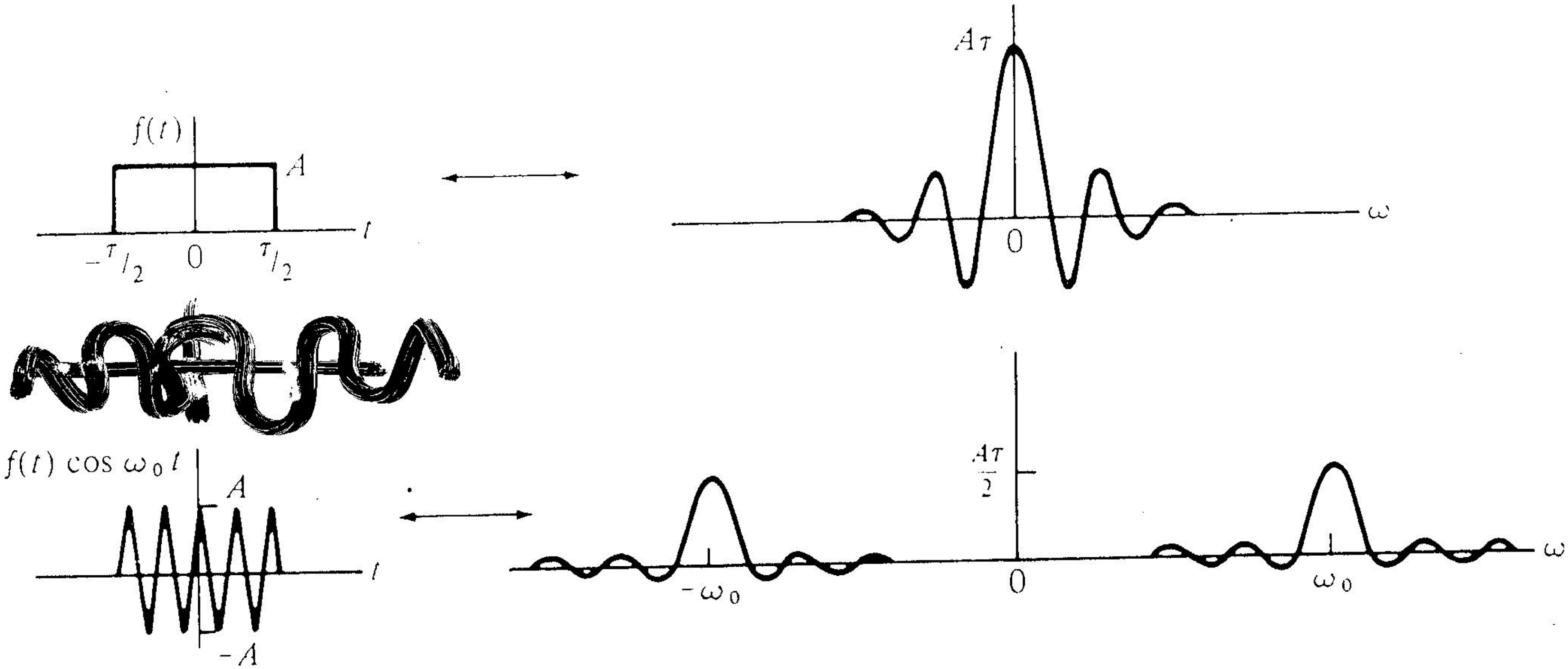

Figure 3.8 Effects of modulation on the signal spectral density.

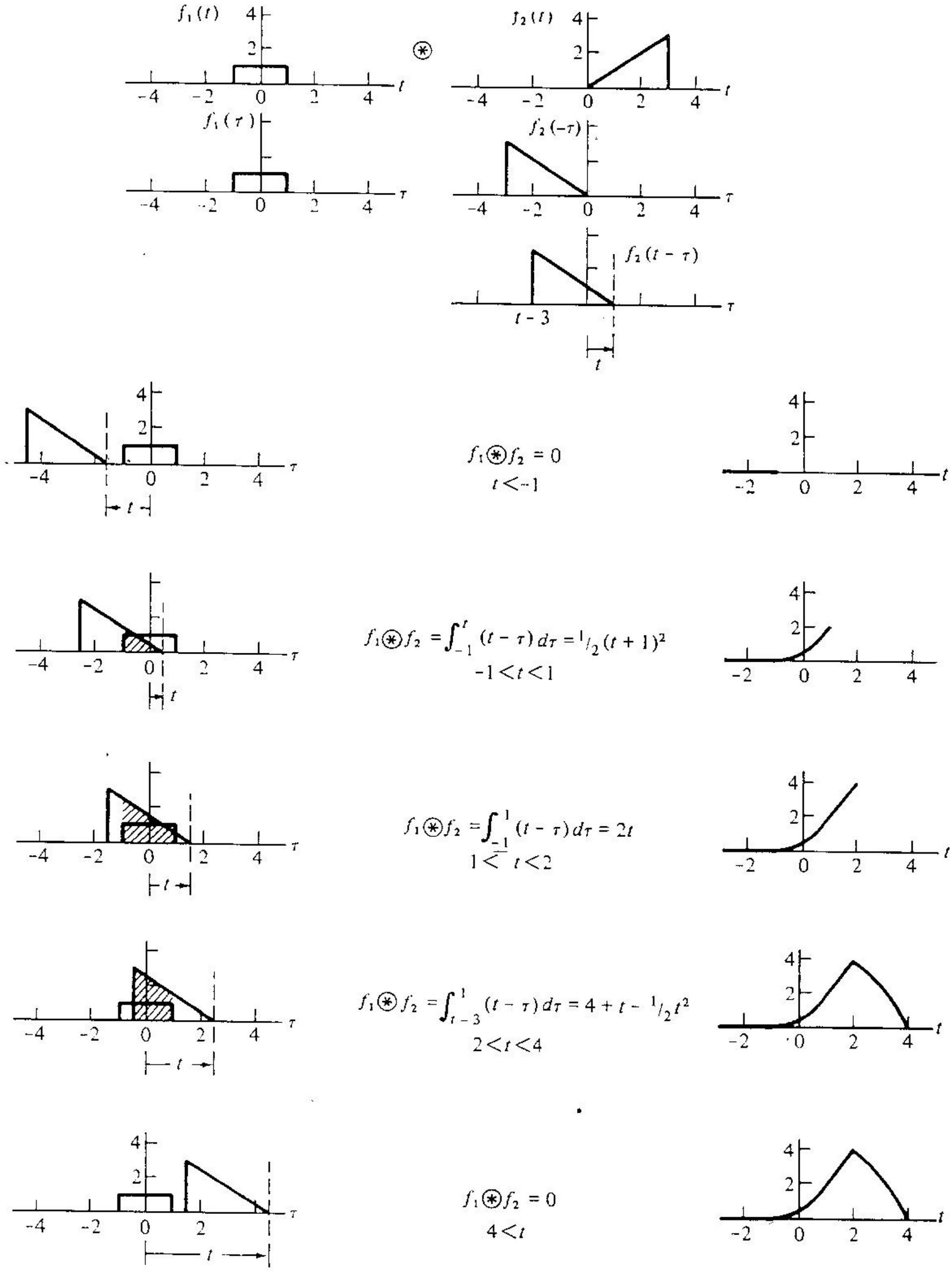

Figure 3.11 The convolution of a rectangular and a triangular pulse.

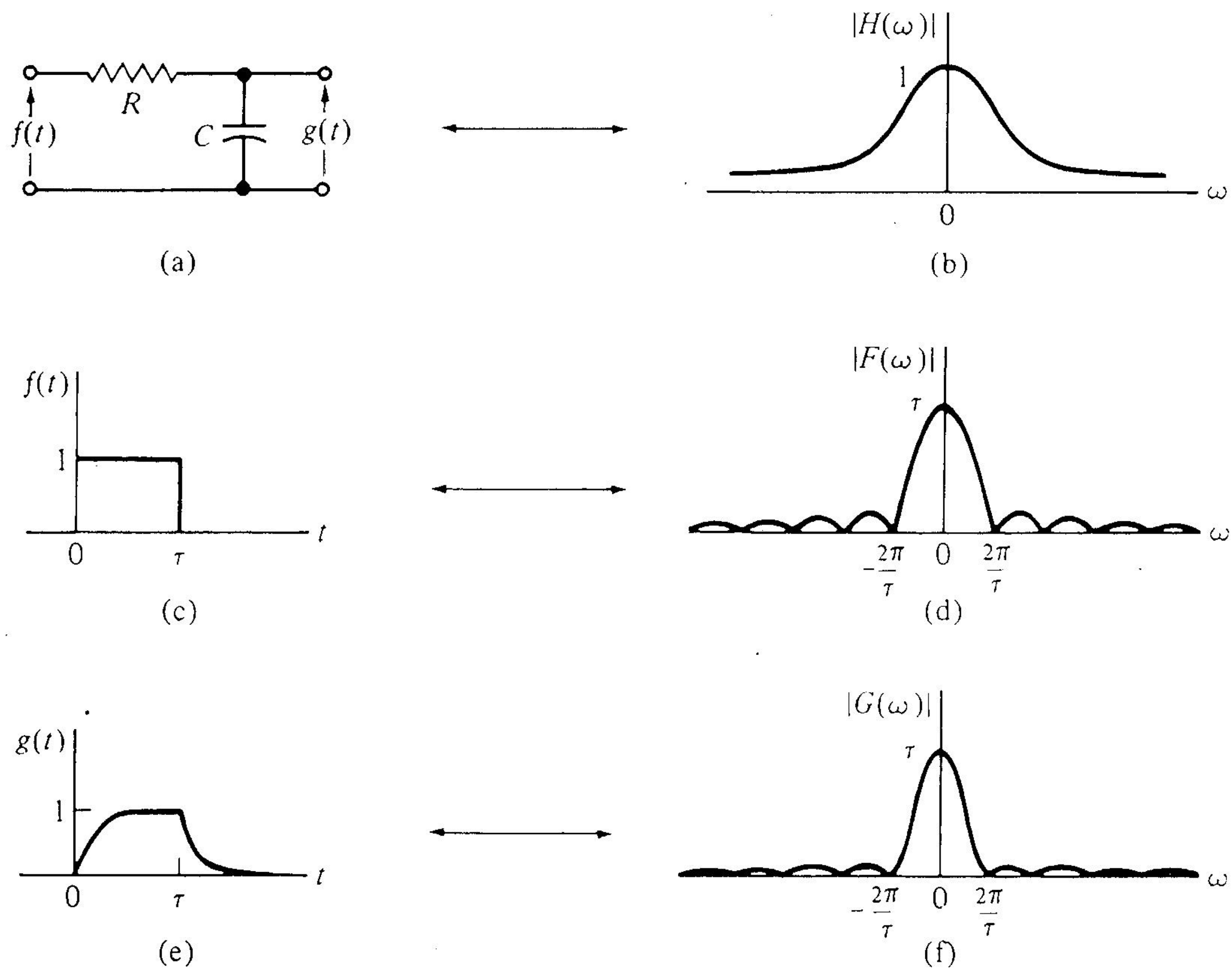

Figure 3.13 Magnitude response of a low-pass filter.

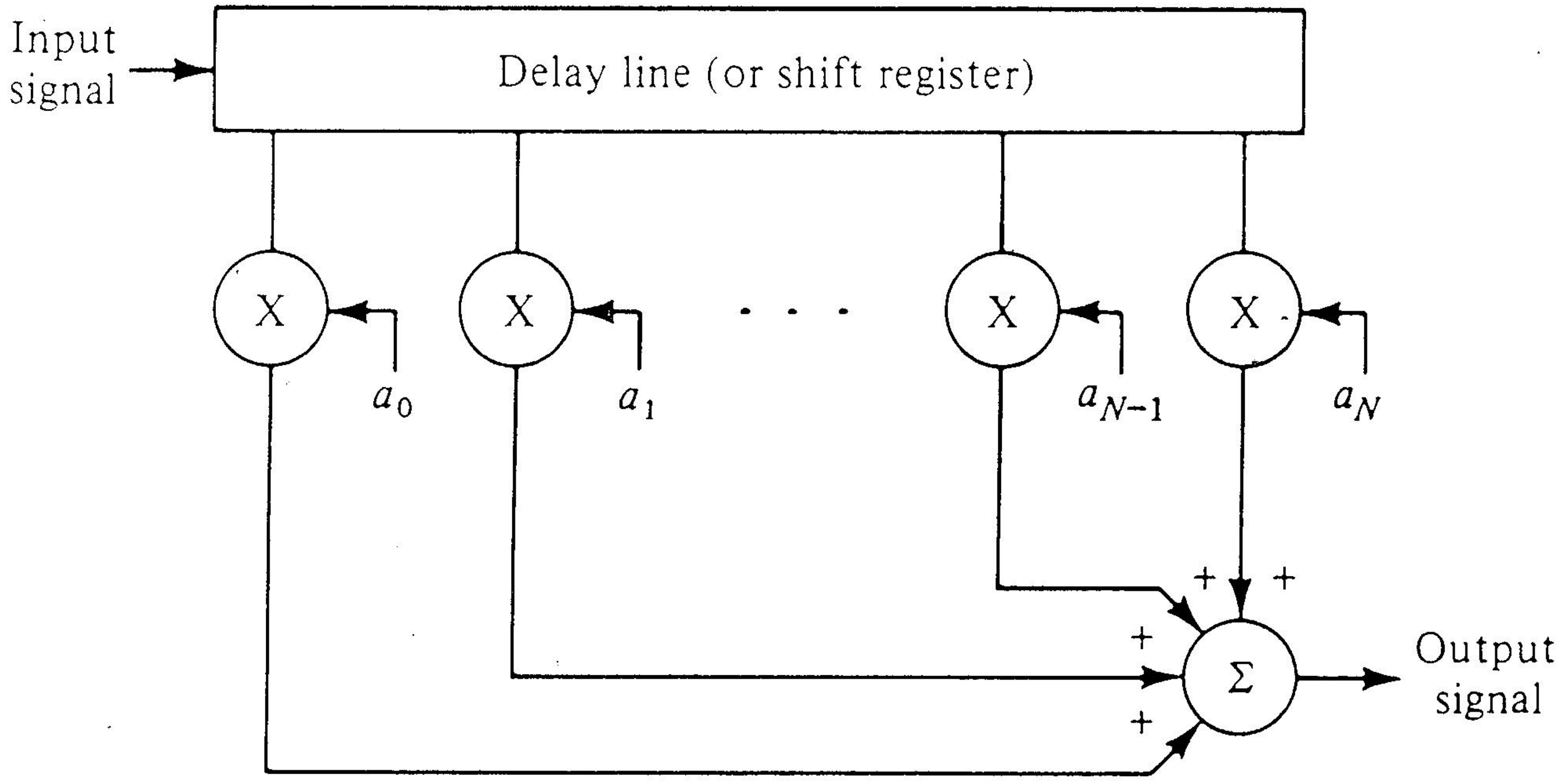

Figure 3.14 A tapped-delay-line (transversal) filter.

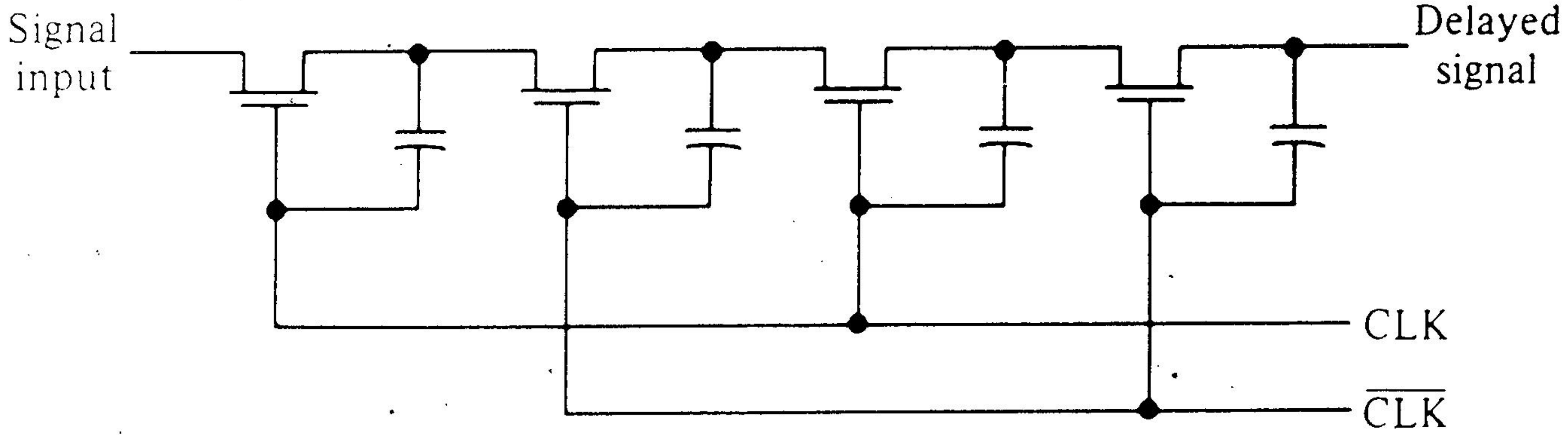

Figure 3.15 A charge-coupled-device (CCD) delay line.

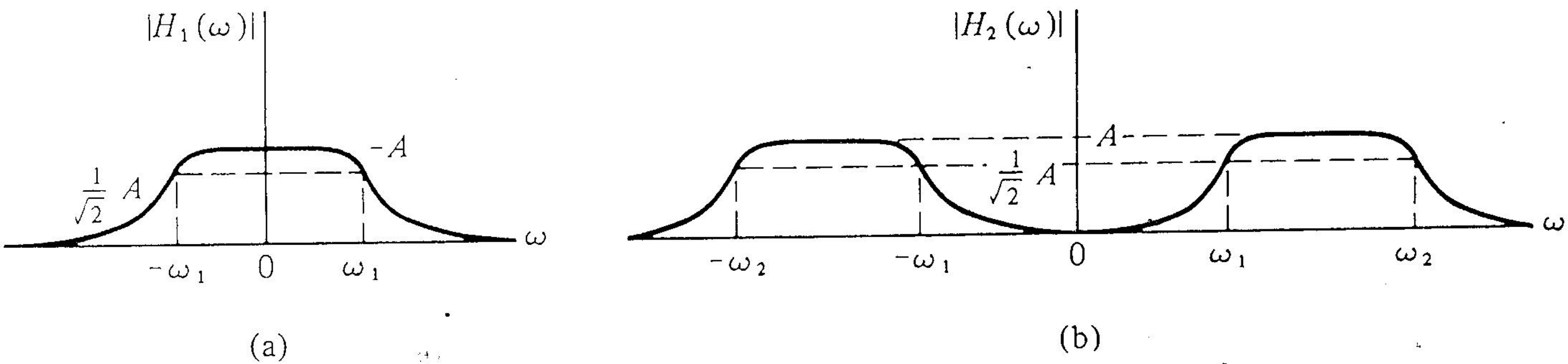

Figure 3.17 The bandwidth of a system as measured to its −3 dB points.

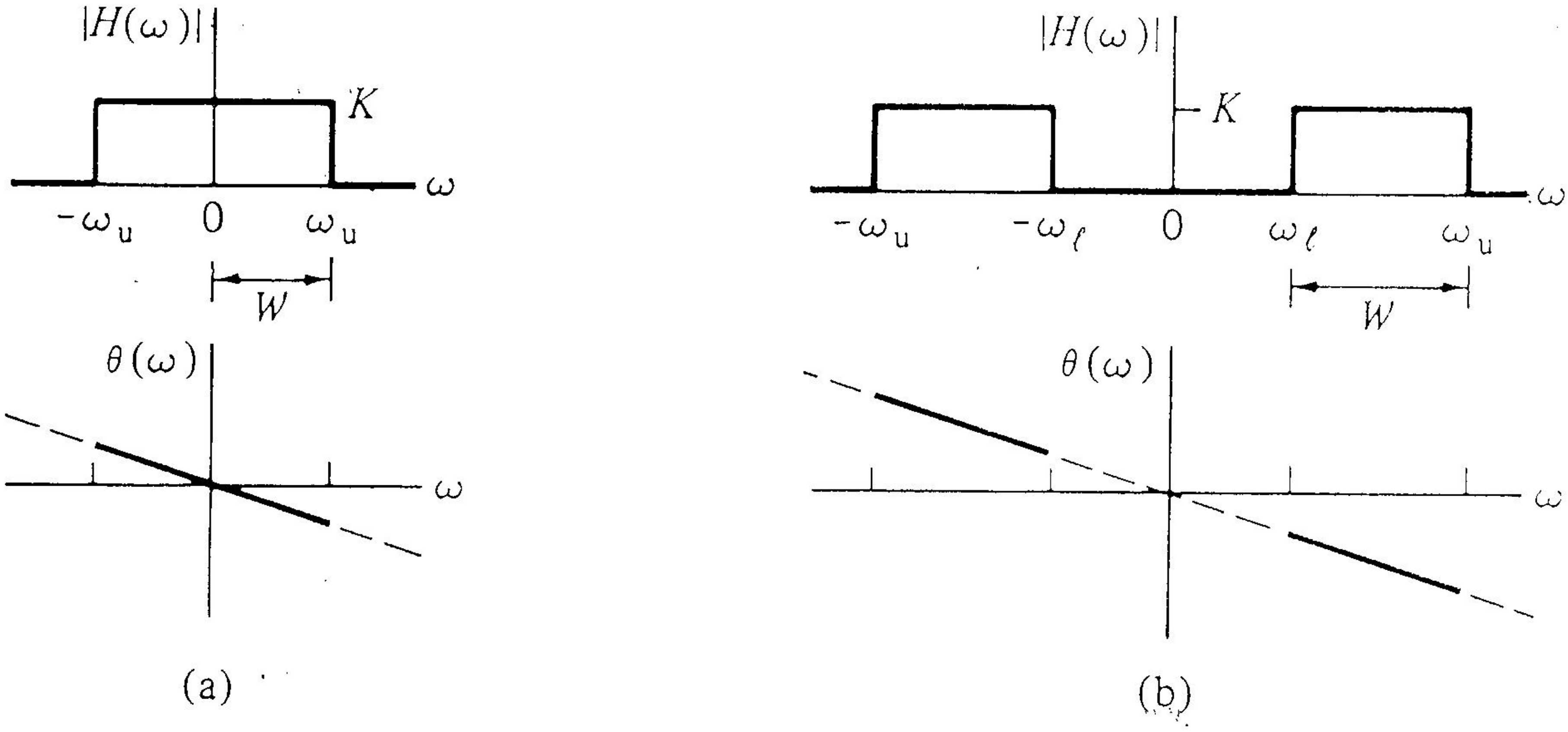

Figure 3.18 The ideal filter: (a) low-pass; (b) bandpass.

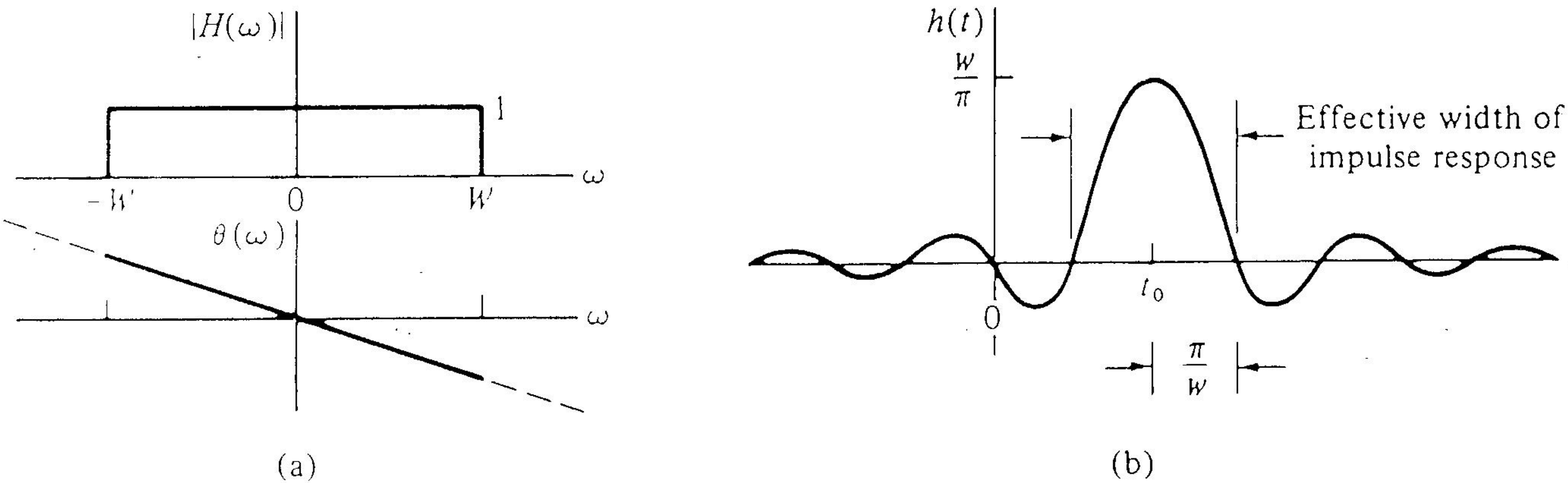

Figure 3.19 The ideal low-pass filter: (a) its system transfer function and (b) its impulse response.

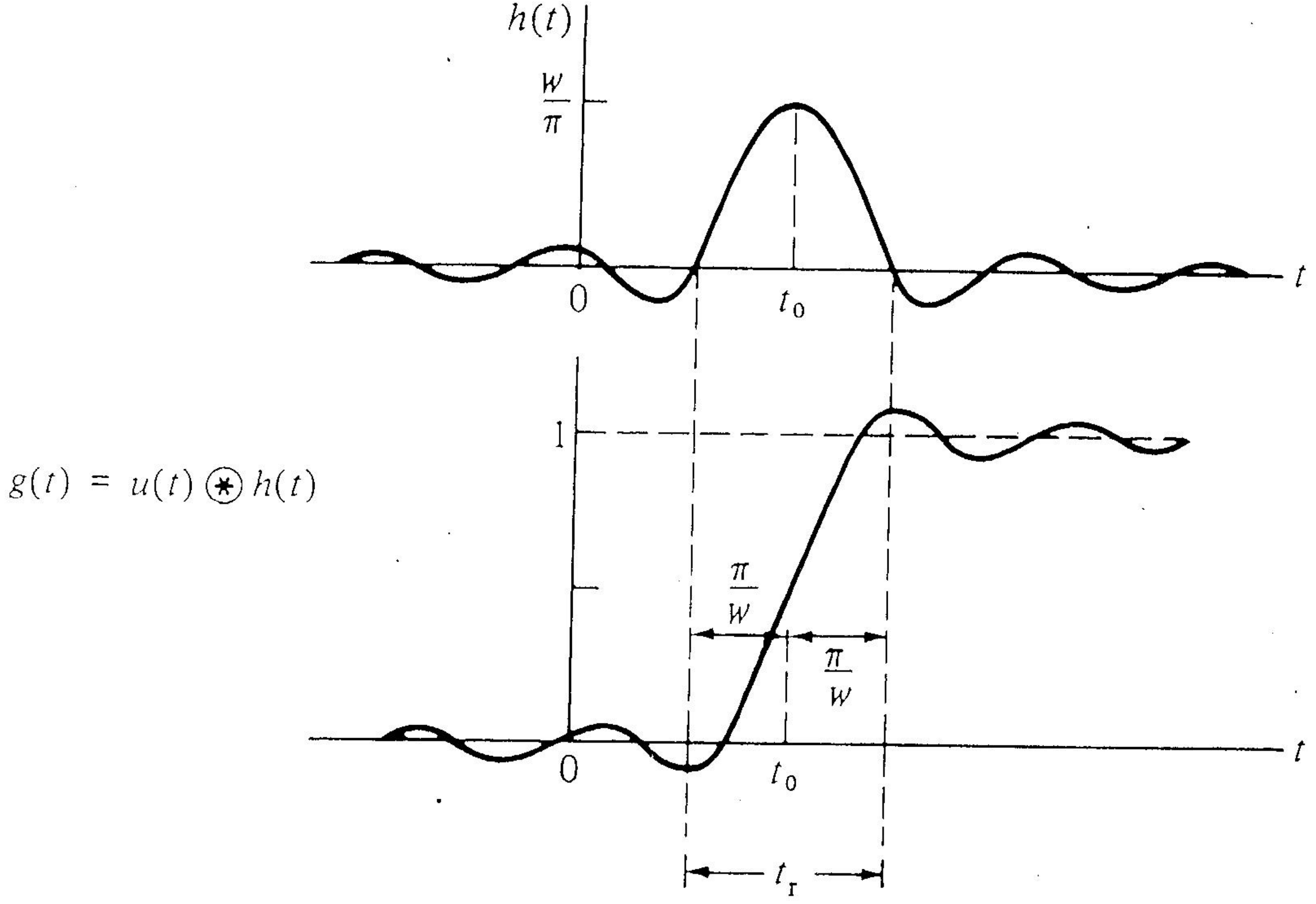

Figure 3.20 The unit step response of an ideal low-pass filter.