Some considerations on investments in irregular dynamic data spaces

J-C. Panayiotopoulos* and P.M. Petrantonakis

Department of Informatics, University of Piraeus, 80, Karaoli & Dimitriou Street, Piraeus 185 34, Greece E-mail: jcp@unipi.gr E-mail: papetr@unipi.gr *Corresponding author

S. Hadjidema

Department of Economics, University of Piraeus, 80, Karaoli & Dimitriou Street, Piraeus 185 34, Greece E-mail: shad@otenet.gr

Abstract: Investing in new products without history is a dangerous task. The investment is even more risky in the case of *irregular dynamic data space* (*ID2S*), where data change within a given planning horizon in an unknown way. Unfortunately, today all data spaces are or tend to become ID2S. Consequently, an optimal solution based on present data is possible to become a complete disaster within our planning horizon. We will not only get a maximum profit, but we will lose and our initial budget too. The present work proposes some new considerations on investment theory based on a new multi-criteria model in order to avoid black investment holes. Also, crisis management and mutative-oriented programming is used.

Keywords: mutative multiobjective mathematical programming; multicriteria decision making under uncertainty; bilevel programming; investment theory; dynamic data space; risk analysis and modelling; crisis management.

Reference to this paper should be made as follows: Panayiotopoulos, J-C., Petrantonakis, P.M. and Hadjidema, S. (xxxx) 'Some considerations on investments in irregular dynamic data spaces', *Int. J. Multicriteria Decision Making*, Vol. X, No. Y, pp.000–000.

Biographical notes: J-C. Panayiotopoulos received his BSc in Maths from the University of Athens in 1970, as well as PhD in Computational Designs in 1975. He runs the Programming and Informatics Management (PIM) Laboratory. He has a wide experience in administrating EU funded projects. He used to be an Assistant Professor in Columbia University (NY) and an Associate Professor in Rutgers (State University of New Jersey). He used to be sub-Rector on Economics and President of the Research Centre of the Piraeus University and also President of the Department of Informatics.

P.M. Petrantonakis received his BSc in Informatics in 2001 and PhD in Crisis Management from the Department of Informatics in University of Piraeus. He worked as a Research Associate (part-time Lecturer) at University of Piraeus.

S. Hadjidema is an Associate Professor at the Department of Economics of the University of Piraeus. She holds a degree in Economics from the University of Athens, an MA in Economics of Public Policy and a PhD in Economics both from the University of Leicester, UK. Her research interests span the field of economics.

1 Introduction

Investments are subject to uncertain future conditions, which are treated as external risk factors over which an investor has no control. In the investment world, risk is inseparable from performance and, rather than being desirable or undesirable, is simply necessary. A common definition for investment risk is deviation from an expected outcome. This 'deviation' can be positive or negative and it is related to the idea of 'no pain, no gain'.

Indeed, in order to achieve higher returns in the long run, we have to accept more short-term volatility. How much volatility depends on the risk tolerance, which is an expression of the capacity to assume volatility based on specific financial circumstances and the propensity to do so, taking into account the psychological comfort with uncertainty and the possibility of incurring large short-term losses.

Investors expect higher returns over time in exchange for holding risky assets, and in a properly diversified portfolio those returns are generally related to the level of risk that they assume. Markets go up and down. It is a fact of life. An appropriate investment strategy anticipates and accounts for market volatility. We know with certainty that bad years are going to happen, even if we do not and cannot know when.

According to the modern financial theory, diversification with assets that have low correlations to one another can be used as a method of optimising a portfolio to provide the maximum expected return per unit of risk that an investor is willing to tolerate (Crouhy et al., 2000). This method does not eliminate risk, guarantees a profit, or prevents loss. However, it has been argued that a lower investment outcome relative to the expected return during any particular period should happen about half the time and thus we can get a general feel for the range of possible outcomes that a portfolio might experience. The timing of events is not known and they are presumed to be random and unpredictable.

These days, *investors face uncertainty from data changing within a planning horizon*. Data change is considered as one of the major risks for investors in the 21st century and policy makers fail to give proper long-term solutions concerning investment choices. A solid understanding of risk in its different forms can help investors to better understand the opportunities, trade-offs and costs involved with different investment approaches in making an optimal or even a good heuristic investment choice.

This paper proposes some new considerations on investment theory in order to avoid the 'black investment holes'. It demonstrates a new multi-criteria model, which could be a useful tool to take into account the data change uncertainty within a planning horizon, thus helping to optimal or good investment decision making. The proposed catastrophe model is supported by an easy to follow example, which clarifies the details of the multi-criteria model. In other words, the present study provides an optimal or even a good solution based on present data, which will continue to be an optimal or a good one in the whole planning horizon, thus facilitating in making optimal investment decisions.

The paper is organised as follows. In Section 2, the investment risk is discussed. Section 3 presents data spaces for investment decision making. Section 4 examines the catastrophe system. The case of operational holes is tackled in Section 5 and a new multi-criteria catastrophe model is described. Finally, our conclusion is presented in Section 6.

2 Investment risk

Investors have a natural aversion to making decisions they will regret. Markowitz (1959), in his groundbreaking work, stated explicitly that investors are risk averse or more concerned about losses than gains. When they look at their risk tolerance, they consider three factors: capacity to take investment risk, need to take investment risk and desire to take investment risk. Measuring risk tolerance is essential to managing expectations for the investor.

One of the most commonly used absolute risk metrics is standard deviation, which explains what happened for the whole period of the investment, but it does not explain what happened along the way. While this information may be helpful, it does not fully address an investor's risk concerns.

What investors really want to know is not just how much an asset deviates from its expected outcome, but how bad things look way down on the left-hand tail of the distribution curve. Value at risk (VAR) attempts to provide an answer to this question. The idea behind VAR is to quantify how bad a loss on an investment could be with a given level of confidence over a defined period of time. Of course, even a measure like VAR does not guarantee that things will not be worse. Spectacular debacles like hedge fund long term capital management (LTCM) in 1998 remind us that so-called 'outlier events' may occur. After all, 95% confidence allows that 5% of the time results may be much worse than what VAR calculates. In the case of LTCM, the outlier event was the Russian government's default on its outstanding sovereign debt obligations, an event that caused the hedge fund's performance to be much worse than its expected VAR.

Other risk measures oriented to behavioural tendencies attempt to address three things: the magnitude of each negative period (how bad), the duration of each (how long) and the frequency (how many times) or measure how comparatively risky is an investment. The field of 'behavioural finance' has contributed an important element to the risk equation, demonstrating asymmetry between how people view gains and losses. In the language of prospect theory, an area of behavioural finance introduced by Kahneman and Tversky (1979), investors exhibit loss aversion, meaning that they put more weight on the pain associated with a loss than the good feeling associated with a gain. More specifically, prospect theory is a behavioural economic theory that describes decisions between alternatives that involve risk, where the probabilities of outcomes are known. The theory says that people make decisions based on the potential value of losses and gains rather than the final outcome, and that people evaluate these losses and gains using interesting heuristics (Kahneman et al., 1982).

More recent studies on choice under uncertainty indicate that people prefer some sources of uncertainty over others. For example, Heath and Tversky (1991) found that

individuals consistently preferred bets on uncertain events in their area of expertise over matched bets on chance devices, although the former are ambiguous and the latter are not. The presence of systematic preferences for some sources of uncertainty calls for different weighting functions for different domains, and suggests that some of these functions lie entirely above others.

Given these findings, one would assume that the task of creating tools to measure risk tolerance accurately would be easy. However, that is far from the case. If the level of market or systematic risk were the only influencing factor, then a portfolio's return would be easily estimated. In fact returns vary as a result of a number of factors unrelated to market risk. Investment managers who follow an active strategy take on other risks to achieve excess returns over the market's performance.

Moreover, cumulative prospect theory, developed by Kahneman and Tversky (1992), attempts to explain the notion that individuals when faced with risky prospects do not make decisions consistent with maximising their benefits. Underweighting of high probabilities contributes both to the prevalence of risk aversion in choices between probable gains and sure things, and to the prevalence of risk seeking in choices between probable and sure losses. Therefore, among other findings, their work confirms that people tend to overweight extreme and therefore low probability events when considering future outcomes. This theory retains the major features of the original version of prospect theory and introduces a (two-part) cumulative functional, which provides a convenient mathematical representation of decision weights. It also relaxes some descriptively inappropriate constraints of expected utility theory. Despite its greater generality, the cumulative functional is unlikely to be accurate in detail. Decision weights may be sensitive to the formulation of the prospects, as well as to the number, the spacing and the level of outcomes.

Some evidence suggests that risk tolerance is also affected by non-emotional factors such as time. In 1992, Camerer (1992) suggested that the curvature of the weighting function is more pronounced when the outcomes are widely spaced. The cumulative prospect theory can be generalised to accommodate such effects, but it is questionable whether the gain in descriptive validity, achieved by giving up the reparability of values and weights, would justify the loss of predictive power and the cost of increased complexity.

Investment programming is based on given data and possibly rules applying in the present time, while the corresponding output has to be reliable within a given programming horizon. It must be noted that only in unreal systems, like computer games, the extreme case of zero programming horizon is valid. Real life is not based on zero programming horizons. For example, an investor may desire a portfolio that has a high standard deviation, but if that investor needs to begin withdrawing money from his portfolio at a very high rate, the volatility of an aggressive portfolio would be deadly to the portfolio's longevity. That relatively small increase in withdrawal rate dramatically increased the probability of achieving a zero value. The reason is the higher volatility primarily in the equity portfolio. The message is, reaching for the small amount of additional return does not pay off, especially when withdrawal rates are relatively high.

Ultimately, the question of risk tolerance can only be answered through a comprehensive review of the investor's financial situation. Risk management tools such as long term care insurance, longer working lives and other factors will significantly affect the investor's ultimate decision regarding asset allocation and portfolio construction. Establishing an understanding regarding risk tolerance is important and, if nothing else, allows the investor to have some idea regarding the decision he or she is making.

3 Data spaces and investment

We denote as $\Delta t = t^2 - t^2$ the whole planning horizon, where t^2 is the end of Δt and t^2 is the present. Investment data can either refer to the static data space (*SDS*) or the dynamic data space (*DDS*). The former does not change within Δ*t*, in other words, the relation $\Delta t = 0$ is true. Optimal or good decisions (optimality/heuristics) in SDS are derived from well known traditional mathematical models. Such spaces soon or later are mutated, unless they are artificial unreal spaces. In the case of DDS, at least one subset of data tends to change within Δ*t* and therefore a crisis might occur.

As far as the DDSs are concerned, they can be either regular dynamic data spaces (*RDDS* or *RD2S*) or irregular dynamic data spaces (*IDDS* or *ID2S*). In RD2S, data change within Δ*t* according to known mathematical rules (i.e., investments in traditional stock markets). Crisis happens only in cases of negligence or incapability. However, "in the case of ID2S, it is not possible to find how data change within a given programming horizon Δ*t*". In such cases *mutative-oriented programming* has to be applied in order to control threats that involve into crisis (Panayiotopoulos et al., 2005; Panayiotopoulos, 2007).

Therefore, "the primal concept in ID2S is that we do not know how data change, but it is possible to know what will cause them to change". It is a set of inactive threads at present, but within Δ*t*, one or more of them may become active catastrophe force and suddenly change the rules of the game. In fact, when a threat is activated in Δt , we get a corresponding catastrophe force, causing total or partial destruction of our original system. The effort to minimise the cost of the disaster and to restore the original status of the system is the core of the well-known 'crisis management'.

threat (*sleeping* $\rangle \rightarrow$ *catastrophe force* (*active*) \rightarrow *crisis*

Figure 1 Organisation of data spaces

Indeed, a DDS is a force field where a potential always exists. If the corresponding potential function is known, then the space is a RD2S, otherwise is an ID2S. Therefore, we need to keep in mind that different aspects of the analysis can pull the design in different directions; one aspect might suggest one structure, while another suggests a different structure. In other words, models employed in RD2S are not sufficient in ID2S.

4 Catastrophe system

The main objective is to find an optimal solution according to present data, which will be an optimal or good one in the whole planning horizon. In other words, if data change within a given programming horizon (i.e., investments in Balkan area, Middle East, East Mediterranean sea, etc.), an optimal solution in the present may not be optimal in the future and also involve into total destruction and instead of resulting in profit, it may damage the initial budget of the investment as well.

In the present study, we attempt to provide an optimal or even a good solution for the whole planning horizon of an investment, based on present data. The question to be answered is "how it is possible in ID2S to find an optimal/good solution at *t*1, which would be optimal/good solution for the entire period $\Delta t = t^2 - t^1$, where $t^2 \geq t^1$.

As it has already been stated, in ID2S an optimal decision at *t*1 might be not optimal at all at *t*2, worst an optimal decision at *t*1 might be a total catastrophe at *t*2. In these spaces we cannot know how data change, but it is possible to know what can change data within Δ*t*. This 'what' is a set of *threats* and factors which at *t*1 are in sleeping situation; that means they do not act upon ID2S at *t*1, but during Δ*t* some of them may be active *catastrophe forces*:

$$
F = \{f_1, f_2, \ldots, f_j, \ldots, f_m\}
$$
, where $j = 1, 2, \ldots, m$.

An active *fj* (i.e., strike, economical crisis, extreme weather events, natural disasters, local wars, unexpected political situation, etc.) may change at least one of the elements of our data space *A* (investment units, funds, etc.):

$$
A = \{a_1, a_2, \ldots, a_i, \ldots, a_n\}
$$
, where $i = 1, 2, \ldots, n$.

Thus, a sleeping threat is becoming an active catastrophe force and this in turn leads to a crisis. Therefore, instead of the original static investment system *S*, we get a new one under IDDS conditions:

$$
S = (A, F, \Delta t)
$$

The average potential *P*(*S*) of the investment system *S* is

$$
P(S) = \left(\sum_{j} \sum_{j} (p_{ij})/m\right) / n \text{ where } i = 1, 2, ..., n, j = 1, 2, ..., m
$$

and $p_{ij} \in [0, 1]$ is the probability associated with the changes of a_i attributed to f_i .

5 Operational holes

We define as alternative point an element a_i of the data space A, which changes its data within Δ*t* due to some crisis of the investment system *S*. Accordingly, we define as alternative set *G*(*S*) a subset of *A*, which contains every alternative point of *A*.

The ultimate goal of our analysis is to make a decision *d*, by taking into consideration only a subset $A(d)$ of the data space A in order to maximise investment profit under a given set of constraints and at the same time minimise the cardinal number of the subset *G*(*A*(*d*)).

Therefore, the *multi-criteria catastrophe model* can be described as follows:

Some considerations on investments in irregular dynamic data spaces 7

maximise
$$
R(X, A(d)) = \sum r(a_i) * x_i, a_i \in A(d)
$$
 (1)

$$
\langle \text{the usual constraints} - \text{if they exist} \rangle \tag{2}
$$

$$
R(X, A(d)) + B(X) \ge R(X^0, A) + B - e
$$
\n(3)

$$
\text{minimise} \left| G(A(d), F, \Delta t) \right| \tag{4}
$$

where

$R(X, A(d)) + B(X)$ is the *mixed financial performance* (*MFP*).

Bilevel programming problems are hierarchical optimisation problems where an objective function is to be maximised over the graph of the solution set mapping of a second parametric optimisation problem (Colson et al., 2007; Bard, 1998). It seems that the mathematical model (1) to (4) can be associated with bilevel programming and specifically with the case of *discrete bilevel programming*. In general, bilevel programming is a very living area, a huge number of questions remain open. These include optimality conditions as well as solution algorithms for problems with non-convex lower level problems, discrete bilevel programming problems in every context, many questions related to the investigation of pessimistic bilevel programming problems, to call only some of them. Also, one implication from NP-hardness, often used in theory, is that such problems should also be solved with approximation algorithms.

In every case the multi-criteria mutative model (1) to (4) runs in exponential time $O(t * 2(n-1))$, where *t* is the simple execution time required for solving a corresponding classical static investment problem. It should be noted that the integer model $[(1)$ to $(4)]$ cannot be solved exactly by any known mathematical method. However, for small values of *n* we can apply a complete enumeration technique finding 'next best solutions' of the sub-model (1) to (2) , until we find a feasible solution of (3) to (4) . On the other hand, it may be possible to find a 'good solution' based on heuristics. Nevertheless, we need to find a new way of modelling in order to solve the ID2S (1) to (4) problem.

Thus, we construct the 0-1 *alternative matrix C*, which is the *map information risk* of the system *S*. The estimation of matrix *C* can be achieved by studying carefully the ID2S, in order to find all threats and the corresponding catastrophe forces (Panayiotopoulos, 2007; Panayiotopoulos and Petrantonakis, 2005a):

$$
C = (c_{ij}), i = 1, 2, ..., n
$$
 and $j = 1, 2, ..., m$

 $c_{ij} = 1$ if the force f_j within Δt can eliminate the profit $r(a_i)$; at worst

may even be phased out and the corresponding capital invested as well,

 $c_{ij} = 0$ otherwise.

A decision *d* made, such that for every $a_i \in A(d)$ the relation $a_i \in G(S)$ is true within Δt [or equivalent, for each $a_i \in A(d)$ there is at least one catastrophe force f_i so that $c_{ij} = 1$], is called *black operational hole* (*BOH*). The negation of BOH is defined as a *white operational hole* (*WHO*), while every other hybrid state is a *gray operational hole* (*GOH*) (Panayiotopoulos, 1992).

At this point, we believe that an easy-to-follow example will clarify the details of the proposed investment model:

Let us select three funds between nine new investment products for one year planning horizon within the ID2S of East Mediterranean Sea with respect to the following data:

 $S = (A, F, \Delta t), m = 7, n = 9,$

 $\Delta t = t^2 - t^2 = 1$ year,

 $e = 11$ money units,

 $budget = 3$ money units / fund, $B = 9$

 $r(a_i) = \{3, 4, 5, 2, 5, 3, 2, 2, 5\}$

Map information risk :

It is obvious that the static optimal solution $A(d^0)$ is:

 $A(d^0) = \{a_3, a_5, a_9\}$

and therefore:

 $R(X^0, A) + B = 5 + 5 + 5 + 3 \cdot 3 = 24$ (pessimistic) minimum acceptable $MFP = 24 - e = 13$

In this case, let us consider the following scenarios within Δ*t*:

- f_2 becomes active $\Rightarrow MFP = 0+0+0+0 = 0$ (BOH: profit and capital are lost...
- f_3 becomes active $\Rightarrow MFP = 5 + 5 + 5 + 9 \ge 24$ (WOH: some competitors may be damaged!)
- *f*₇ becomes active $\Rightarrow MFP = 5 + 0 + 5 + 6 = 16$ (GOH)
- *f*₁ becomes active $\Rightarrow MFP = 5 + 0 + 0 + 3 = 8$ (GOH).

Consequently, it is necessary to find next best solutions until to find one with a worst case (pessimistic case) of $MFP \geq 13$. In our example, the dynamic optimal solution $A(d)$ is:

$$
A(d) = \{a_1, a_2, a_3\}
$$

with a pessimistic $MFP = 3 + 4 + 0 + 6 = 13$ and optimistic $MFP = 3 + 4 + 5 + 9 = 21$.

In the above analysis, it was assumed that only one catastrophe force f_i appears within Δ*t* (Panayiotopoulos and Petrantonakis, 2005b). This is called 'crisis of order 1'. In general, in a case of *k* active forces within Δ*t*, there is a 'crisis of order *k*'. For instance, in our example the dynamic optimal solution $A(d) = \{a_1, a_2, a_3\}$ is going to be a BOH (crisis of order 3), if some threats energised the forces f_1, f_2, f_3 within the planning horizon Δt .

6 Conclusions and further investigations

Employing theories of choice for investment considerations, we realise that they are at best approximate and incomplete. One reason for this pessimistic assessment is that choice is a constructive and contingent process. When faced with a complex problem, people employ a variety of heuristic procedures in order to simplify the representation and the evaluation of prospects. These procedures include computational shortcuts and editing operations, such as eliminating common components and discarding non-essential differences (Tversky, 1969). The heuristics of choice do not readily lend themselves to formal analysis, because their application depends on the formulation of the problem, the method of elicitation, and the context of choice.

Recent studies on investments in DDSs are concerned with RD2S. They all focus on risk tolerance and they mainly suggest that diversification with assets that have low correlations to one another can be used as a method of optimising a portfolio. Therefore, investors should try to divide the investments among the least correlated assets in order to get the maximum expected return per unit of risk that they are willing to tolerate. However, the timing of events is not known and they are presumed to be random and unpredictable. *Today, most of the economic spaces are (or will be) irregular ones, meaning that data change within a planning horizon according to an unknown way*. It seems that the conjunctive "the existence of the unexpected for each system is certainty" is true. This is not a pessimistic idea. We just think that many classical notions of micro-economics and operations research should be replaced due to fuzzy nature of ID2Ss.

A 'good solution' is to avoid investments that can lead to BOH or GOH. The secret is to take into consideration *countermeasures* before any crisis (Panayiotopoulos et al., 2005). Therefore, to minimise investment risks, policy-makers should aim to provide a good technique, that is to choose products which are different between them concerning

the catastrophe system *S*. They have to take into consideration products with the maximum average difference of catastrophe, i.e. the average probability to be catastrophe points will be the minimum possible.

In our analysis, we considered only the case where the risk probabilities are equal to 1 or 0. The assumption made in this comprehensive model is not a strong one. In real world problems, it is almost impossible to know precisely the values of p_{ii} (Panayiotopoulos and Petrantonakis, 2005b, 2006).

Finally, although this seems to be a 'good solution' for optimal decision-making, investing in ID2S still remains an open question. Therefore, the investigation of investments in ID2S's emerges as a promising domain for future research.

References

- Bard, J.F. (1998) *Practical Bilevel Optimization: Applications and Algorithms*, Kluwer Academic Press, Dordrecht, Netherlands.
- Camerer, C.F. (1992) 'The rationality of prices and volume in experimental markets', *Organizational Behavior and Human Decision Processes*, Vol. 51, pp.237–272.
- Colson, B., Marcotte, P. and Savard, G. (2007) 'An overview of bilevel optimization', *Ann. Oper. Res*., Vol. 153, pp.235–256, DOI 10.1007/s10479-007-0176-2.
- Crouhy, M., Galai, D. and Mark, R. (2000) 'A comparative analysis of current credit risk models', *Journal of Banking and Finance*, Vol. 24, Nos. 1–2, pp.59–117.
- Heath, C. and Tversky, A. (1991) 'Preference and belief: ambiguity and competence in choice under uncertainty', *Journal of Risk and Uncertainty*, Vol. 4, pp.5–28.
- Kahneman, D. and Tversky, A. (1979) 'Prospect theory: an analysis of decision under risk', *Econometrica*, Vol. 47, No. 2, 263p.
- Kahneman, D. and Tversky, A. (1992) 'Advances in prospect theory: cumulative representation of uncertainty', *Journal of Risk and Uncertainty*, Vol. 5, pp.297–323.
- Kahneman, D., Slovic, P. and Tversky, A. (1982) *Judgment Under Uncertainty: Heuristics and Biases*, Cambridge University Press, New York.
- Markowitz, H.M. (1959) *Portfolio Selection: Efficient Diversification of Investments*, John Wiley & Sons, New York (reprinted by Yale University Press, 1970; 2nd ed., Basil Blackwell, 1991).
- Panayiotopoulos, J-C. (1992) 'White, gray and black operational holes: an artificial intelligence approach', *Information and Optimization Sciences*, Vol. 13, No. 3, pp.407–425.
- Panayiotopoulos, J-C. (2007) 'Information operations/crisis management in cyberspace: mutative oriented programming', *Crisis Management International Conference*, 4–7 July 2007, Athens.
- Panayiotopoulos, J-C. and Petrantonakis, P. (2005a) 'Minimizing recovery costs in military information systems: the land warrior case', *Defensor Pacis*, Vol. 17, pp.189–200.
- Panayiotopoulos, J-C. and Petrantonakis, P. (2005b) 'Using assignment model as an automated recovery system', *The International Journal of Disaster Prevention and Management*, Vol. 14, No. 1, pp.89–97.
- Panayiotopoulos, J-C. and Petrantonakis, P. (2006) 'Some considerations on crisis management', *Defensor Pacis*, Vol. 18, pp.7–14.
- Panayiotopoulos, J-C., Mavrommatis, G. and Kalligatsis, J. (2005) 'MOP: mutation-oriented programming', *Foundations of Computing and Decision Sciences*, Vol. 30, No. 3, pp.215–226.
- Tversky, Α. (1969) 'Intransitivity of preferences', *Psychological Review*, Vol. 76, pp.31–48.