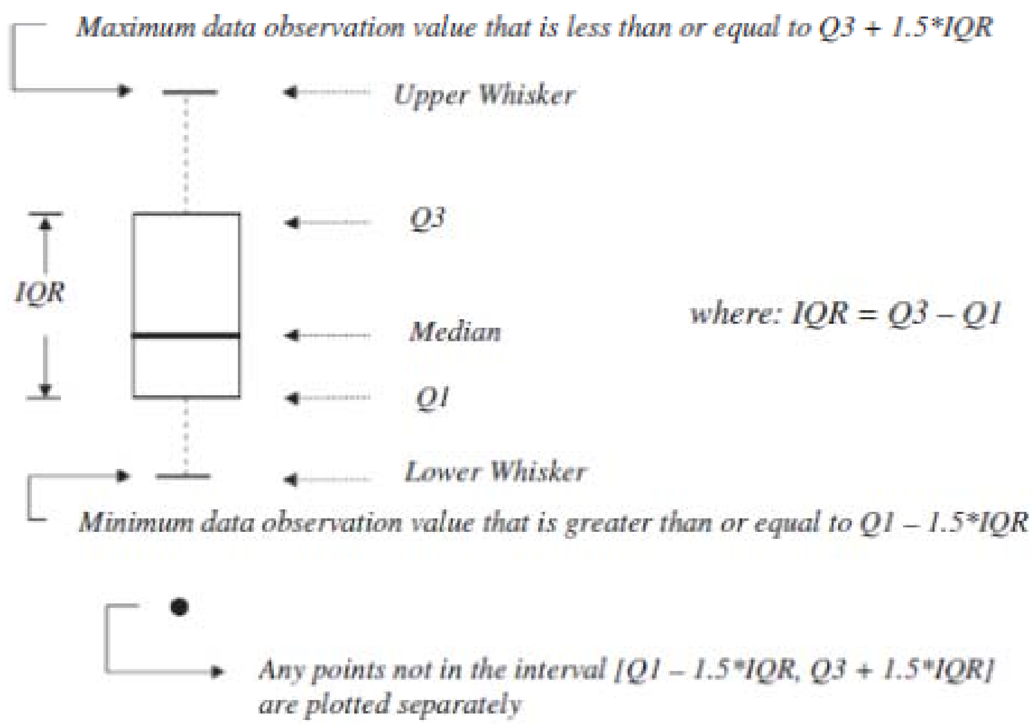

Boxplot's basic functionality.

5.1. *Hypervolume*

As shown in Table 7 and Fig. 13 the most popular performance metric in the field is the hypervolume (HV).[24,25,47,63,64] HV, also known as Lebesgue measure,[45] S metric[74] or 'hyperarea metric'[67] is an indicator of both the convergence and diversity of an approximation set. Thus, given a set S containing m points in n objectives, the HV of S is the size of the portion of objective space that is dominated by at least one point in S (see Fig. 14).

As reference point is taken the worst known value in each objective.

The HV of S is calculated relative to a reference point which is worse than (or equal to) every point in S in every objective. The greater the HV of a solution the better considered the solution. One of the main advantages of HV[78] is that it is able to capture in a single number both the closeness of the solutions to the optimal set and, to some extent, the spread of the solutions across objective space. According to a

Fig. 14. HV of a bi-objective minimization problem.

number of studies, HV also has nicer mathematical properties than many other metrics. In particular, Zitzler *et al.*[79] state that HV is the only unary metric of which they are aware that is capable of detecting that a set of solutions X is not worse than another set X'.

5.2. *Epsilon indicator*

Epsilon indicator is the second most popular performance metric as it is indicated by the studies in the field.[1–3,47] Zitzler *et al.*[79] introduced the epsilon indicator. There are two versions of epsilon indicator the multiplicative (I_{ε}) and the additive ($I_{\varepsilon+}$). The basic usefulness of epsilon indicator of an approximation set A is that it provides the minimum factor ε or respectively the minimum term ε by which each point in the real front R can be multiplied or respectively added, such that the resulting transformed approximation set is dominated by A. The epsilon indicator is a good measure of diversity, since it focuses on the worst case distance and reveals whether or not the approximation set has gaps in its trade-off solution set. The smaller the epsilon indicator, the better is the approximation set. The Pareto-optimal solutions set has $I_{\varepsilon} = 1$ and $I_{\varepsilon+} = 0$.

The epsilon indicator I_{ε} is defined for two Pareto sets A and B as $I_{\varepsilon}(A, B) = \inf_{\varepsilon \in R}\{\forall z^2 \in B \exists z^1 \in A : z^1 \preccurlyeq_{\varepsilon} z^2\}$, where in the case of a minimization problem the multiplicative ε-dominance relation is defined as: $z^1 \preccurlyeq_{\varepsilon} z^2 \iff \forall i \in 1, \ldots, n : z_i^1 \preccurlyeq \varepsilon \cdot z_i^2$. The additive epsilon indicator $I_{\varepsilon+}$ is defined by using the additive ε-dominance: $z^1 \preccurlyeq_{\varepsilon} z^2 \iff \forall\, i \in 1, \ldots, n : z_i^1 \preccurlyeq \varepsilon + z_i^2$.

Many studies in the field use both HV and Epsilon indicator for the evaluation of the relevant results. According to Knowles *et al.*[40] when two algorithms generate conflicting preferences between these two metrics they are incomparable.

5.2. Epsilon indicator

Epsilon indicator is the second most popular performance metric as it is indicated by the studies in the field (Anagnostopoulos & Mamanis, 2010, 2011a, 2011b; Liagkouras & Metaxiotis, 2014). Zitzler et al. (2003) introduced the epsilon indicator. There are two versions of epsilon indicator the multiplicative (I_ε) and the additive ($I_{\varepsilon+}$). The basic usefulness of epsilon indicator of an approximation set A is that it provides the minimum factor ε or respectively the minimum term ε by which each point in the real front R can be multiplied or respectively added, such that the resulting transformed approximation set is dominated by A. The epsilon indicator is a good measure of diversity, since it focuses on the worst case distance and reveals whether or not the approximation set has gaps in its trade-off solution set. The smaller the epsilon indicator, the better is the approximation set. The Pareto-optimal solutions set has $I_\varepsilon = 1$ and $I\varepsilon_+ = 0$.

The epsilon indicator I_ε is defined for two Pareto sets A and B as $I_\varepsilon(A, B) = inf_{\varepsilon \in \mathbb{R}}\{\forall z^2 \in B \exists z^1 \in A{:}z^1 \preceq_\varepsilon z^2\}$. Where in the case of a minimization problem the multiplicative ε-dominance relation is defined as: $z^1 \preceq_\varepsilon z^2 \Leftrightarrow \forall i \in 1, \ldots, n : z_i^1 \preceq \varepsilon + z_i^2$. The additive epsilon indicator $I_{\varepsilon+}$ is defined by using the additive ε-dominance: $z^1 \preceq_\varepsilon z^2 \Leftrightarrow \forall i \in 1,, n : z_i^1 \preceq \varepsilon + z_i^2$.

5.3.3 *Epsilon indicator* (I_ε)

There are two versions of epsilon indicator the multiplicative and the additive (Zitzler et al. 2000). In this study we use the unary additive epsilon indicator. The epsilon indicator of an approximation set A ($I_{\varepsilon+}$) provides the minimum factor ε by which each point in the real front R can be added such that the resulting transformed approximation set is dominated by A. The additive epsilon indicator is a good measure of diversity, since it focuses on the worst case distance and reveals whether or not the approximation set has gaps in its trade-off solution set.

Inverted Generational Distance (IGD)

According to Riquelme et al. [57] Inverted Generational Distance (IGD) is the fourth most popular performance metric, based on number of citations by relevant studies in the field. The Inverted Generational Distance (IGD) was proposed as an improvement over the generational distance (GD) [61]. In particular the IGD measures

the closeness of the obtained solution set to the true Pareto optimal set and is given by the following relationship:

$$\mathrm{IGD(P, S)} = \frac{(\sum_{i=1}^{|P|} d_i^q)^{1/q}}{|P|}$$

where $d_i = min_{\vec{s} \in S} \left\| F(\vec{p}_i) - F(\vec{s}) \right\|, \vec{p}_i \in P, q = 2$ *and* d_i is the smallest distance of $\vec{p} \in P$ to the closest solutions in S. When the Pareto optimal set is not known the IGD can be estimated based

on approximations of the Pareto optimal set.