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Network Dynamics, pt. I
Learning in Networks

Session Objectives

- **Explain the role of networks in opinion formation**
  - Discuss the importance of opinion leaders
- **Build alternative opinion formation models**
  - Consider different assumptions concerning agents' rationality
  - Bounded rationality (**naïve learning**)
- **Address fundamental questions on opinion formation**
  - Convergence and consensus of opinions
  - Which individuals have most influence over beliefs
  - How quickly do individuals learn
  - Whether diverse information can be accurately aggregated in the long-run (i.e. whether **asymptotic learning** takes place)

Session Outline

- **Early Theory and Opinion Leaders**
  - Information sharing and opinion formation
  - Influencers and opinion leaders in social networks
- **Imitation and Social Influence models**
  - The De Groot model
  - Incorporating media and opinion leaders
- **Convergence and Consensus**
  - Convergence in the De Groot Model
  - Consensus in the De Groot Model
- **Social Influence**
  - Measuring influence in the De Groot Model
  - Influence in Krackhardt's advice network
- **Rate and Accuracy of Convergence**
  - Rate of convergence in the De Groot Model
  - Accuracy of convergence: The wisdom of crowds

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Early Theory and Opinion Leaders

Information sharing and opinion formation
Influencers and opinion leaders in social networks

Social networks: information sharing and opinion formation

Social networks have a central role in information sharing and opinion formation

Examples: Advising friends on which movies to see, relaying information about a potential new employee, debating about politics, providing information on scientific research and results.

Diffusion of information, opinion formation and subsequent behavior of agents depends on the network structure

Information sharing and opinion formation

Fundamental questions

- Individuals, within a society, come to hold a **common belief** or remain divided in their opinions?
- Who are the individuals who have the **most influence** over others' beliefs within a society?
- **How quickly** do individuals learn?
- Can initially diverse information, scattered throughout the society, be **aggregated in an accurate manner**?

Opinion Leaders and Influencers

Early Literature on Opinion Leaders

- Lazarsfeld, Berelson, Gaudet (1944) - **identifying opinion leaders** through observing individual voting decisions:
  Opinion leaders are individuals who **receive information through various media and interactions**, form their opinion and then **convey it to others who have less direct information**.

- Picked up by Katz and Lazarsfeld (1955):
  - While sometimes opinion leaders hold **higher social status**, in many cases they are of the **same status as those whom they influence**, especially with regard to household decisions.
  - Often distinguished by their **popularity** and the **size of their families**.

- Katz, E. and Lazarsfeld, P.F. (1955). *Personal Influence: The Part Played by People in the Flow of Mass Communication*, The Free Press, New York.
- Lazarsfeld, P.F., Berelson, B. and Gaudet, H. (1944). *The People's Choice: How the Voter Makes up His Mind in a Presidential Campaign*, Columbia University Press, New York.

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Imitation and Social Influence Models

The De Groot model

Incorporating media and opinion leaders

Imitation and Social Influence Models: DeGroot (1974)

DeGroot model: useful as a starting point to understand **how the structure of a network influences the spread of information and opinion formation.**

DeGroot model: A model of Bounded Rationality

- Agents communicate in a network.
- Each agent repeatedly takes the (weighted) average beliefs of her neighbors and re-assesses her own opinion based on these.
- The process repeats in an infinite number of steps.

Notes:

- **The model is not Bayesian (fully rational)**, since agents do not adjust the weights they place on their neighbors' opinions over time.

DeGroot, M.H. (1974), Reaching a consensus, *Journal of the American Statistical Association*, vol. 69, pp. 118-121.

Imitation and Social Influence Models: DeGroot (1974)

Modeling Framework

- **Individuals** {1, ..., n}

- **Influence Matrix:** A weighted and directed n x n non-negative matrix T (i.e., a **row stochastic** matrix whose entries across each row sum to one).
- **T_{ij}**: the **weight or trust that agent i places on the current belief of agent j** in forming her belief for the next period.

- Each agent starts with a set of **beliefs $p_i(0)$ in [0,1].**
- Beliefs are updated over time:

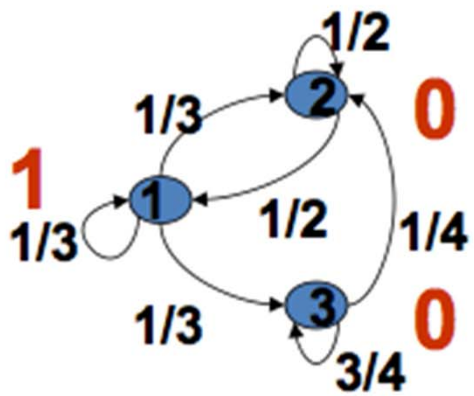
$$p_i(t) = \sum_j T_{ij} p_j(t-1)$$

Imitation and Social Influence Models: DeGroot (1974)

An example of belief updating in the DeGroot model

3 individuals, influence matrix T

- Agent 1 weights all beliefs (including own) equally.
- Agent 2 weights agents 1 and 2 equally and ignores agent 3.
- Agent 3 weights 2 and 3 and ignores 1; Agent 3 places more weight on own belief.



$$T = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 1/2 & 0 \\ 0 & 1/4 & 3/4 \end{pmatrix}$$

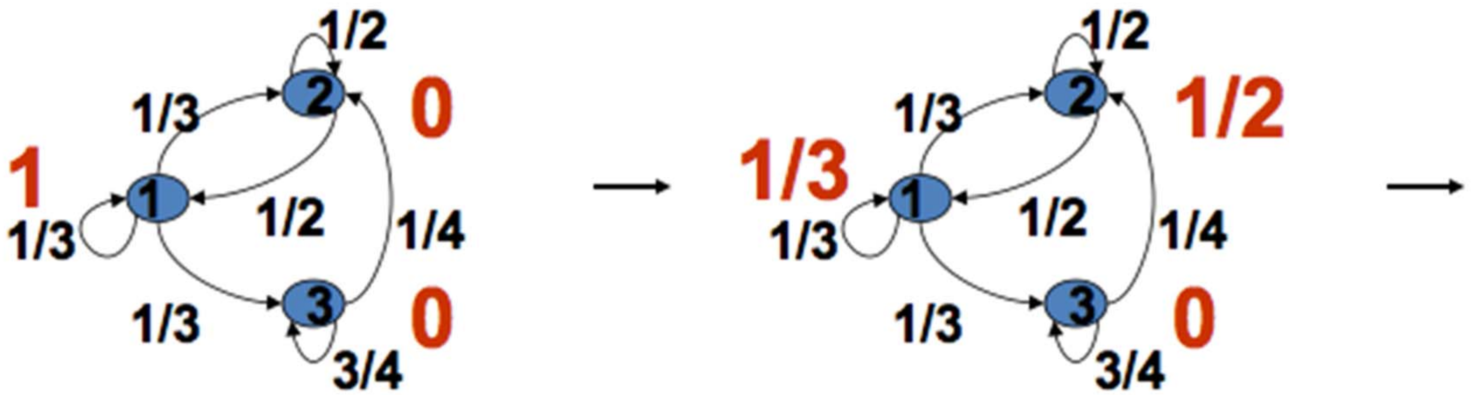
(Jackson, M.O., 2013)

Imitation and Social Influence Models: DeGroot (1974)

- Let's assume that agent 1 starts with a belief of 1, while agents 2 and 3 start with a belief of 0:

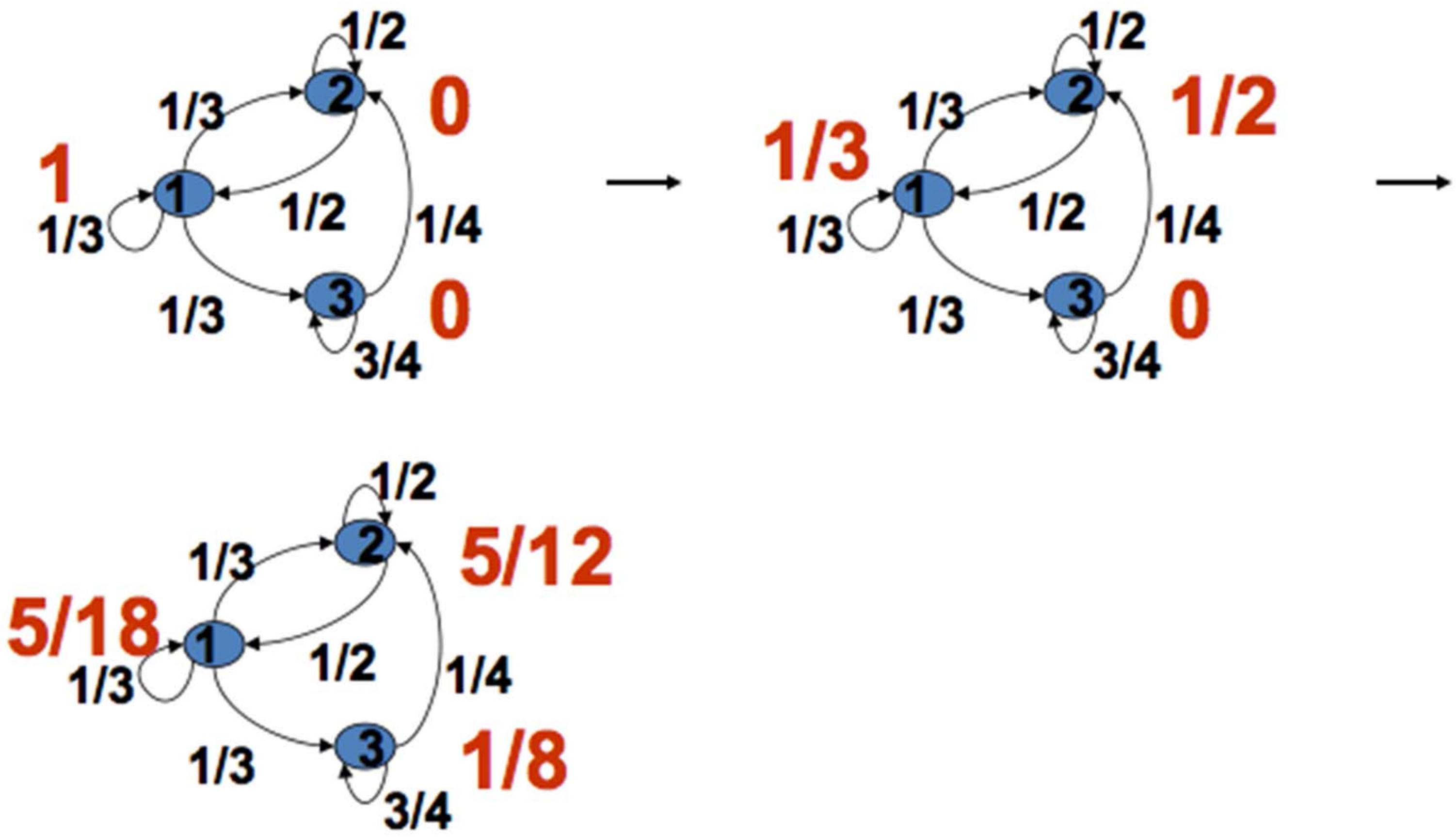
$$p(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

- Update: $p(1) = Tp(0) = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 1/2 & 0 \\ 0 & 1/4 & 3/4 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 1/2 \\ 0 \end{pmatrix}$



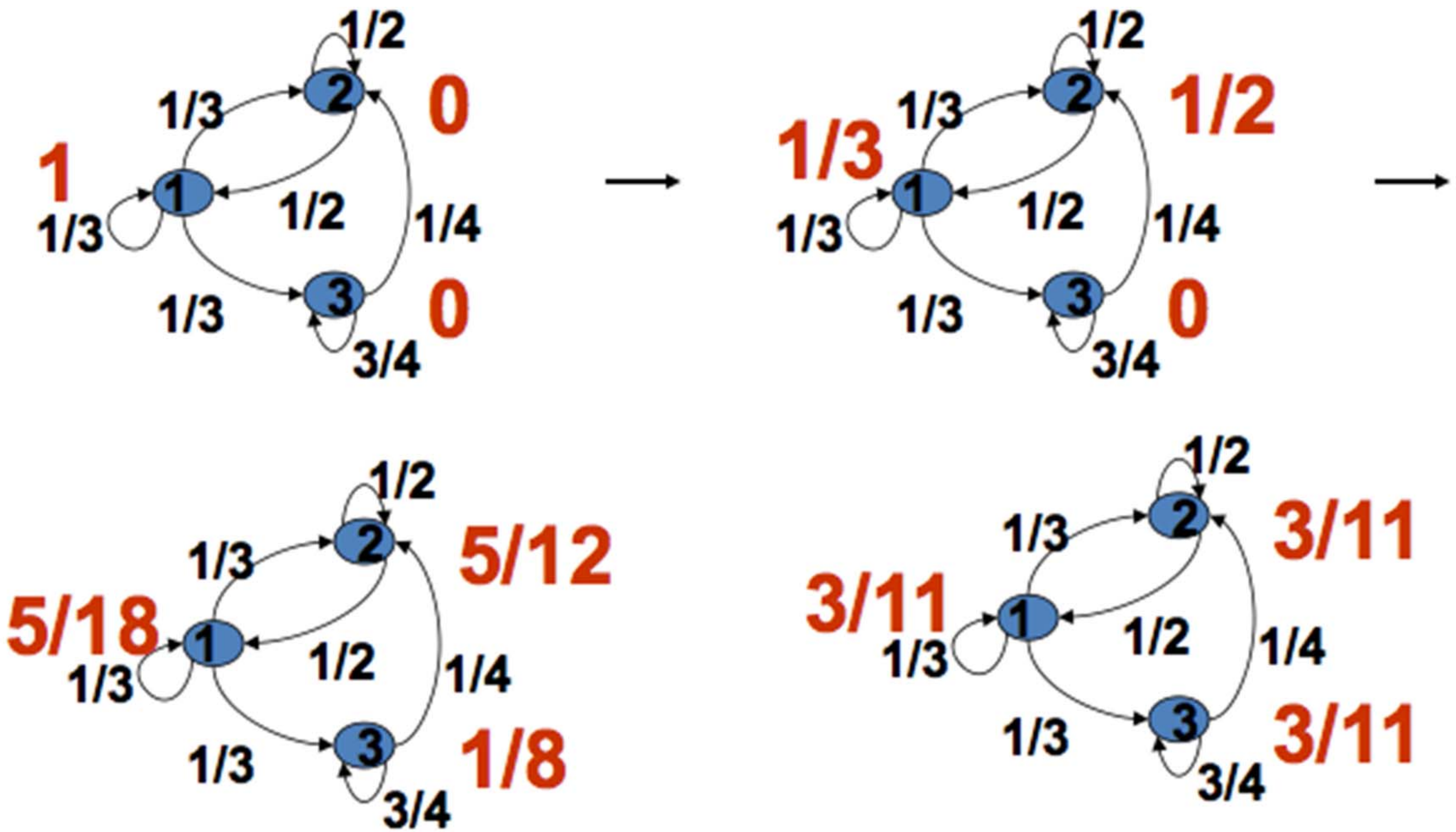
Imitation and Social Influence Models: DeGroot (1974)

- Agents update again: $p(2) = Tp(1) = \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/2 & 1/2 & 0 \\ 0 & 1/4 & 3/4 \end{pmatrix} \begin{pmatrix} 1/3 \\ 1/2 \\ 0 \end{pmatrix} = \begin{pmatrix} 5/18 \\ 5/12 \\ 1/8 \end{pmatrix}$



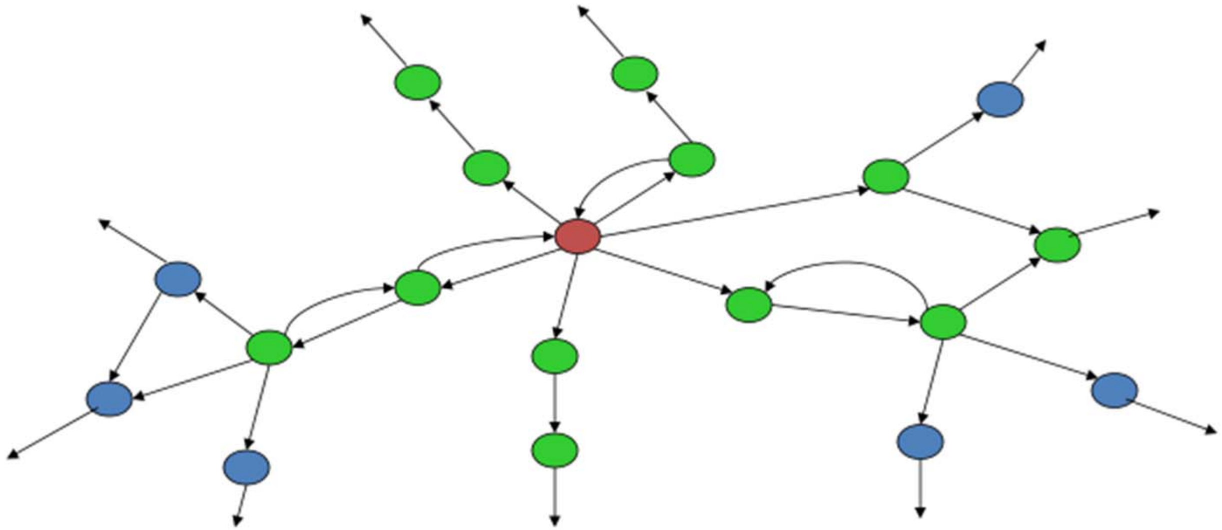
Imitation and Social Influence Models: DeGroot (1974)

- Iteration leads to: $p(t) = Tp(t-1) = T^t p(0) \rightarrow \begin{pmatrix} 3/11 \\ 3/11 \\ 3/11 \end{pmatrix}$



Imitation and Social Influence Models: DeGroot (1974)

- Iteration allows the **incorporation of more distant information** and allows the network (possibly) to reach **consensus**.
- However, the model is a **boundedly rational** version of reality, with no adjustment.
- Still, there are situations where updating according to this very simple process will still lead agents to converge to a fully **accurate belief in the limit**.



Incorporating media and opinion leaders

How do external sources of information (e.g. opinion leaders) influence a society?

- **The DeGroot model can incorporate external information providers**, who are not influenced by others but listened to.
- External sources introduced as i's with **T_{ii} = 1 and T_{ij} = 0 for j≠i when T_{ji} > 0 for some j's** (i.e., modeled as an agent i whose opinion stays fixed at $p_i(0)$, but whom other nodes pay attention to)

- **Opinion leaders** arise naturally in the model, having non-negligible influence.
- The influence of agent j over others' final beliefs will **depend on how much weight other individuals place on the agent**.

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Convergence and Consensus

Convergence in the De Groot Model
Consensus in the De Groot Model

Convergence in DeGroot model

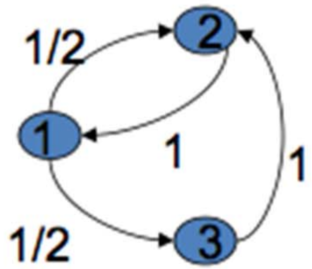
- **Under what conditions will the updating process converge to a well-defined limit?**

- **What limit does the process converge to?**

A social influence matrix T is convergent if $\lim_{t \to \infty} T^t p(0)$ **exists for all initial vectors of beliefs p(0).**

- The process of convergence is illustrated in the **example** below.

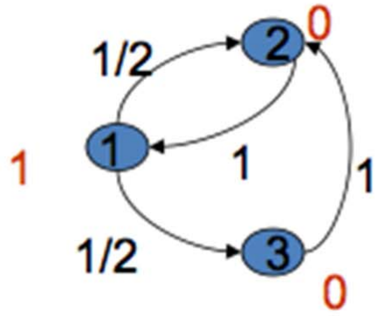
Convergence in DeGroot model



$$T=\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

(Jackson, M.O., 2013)

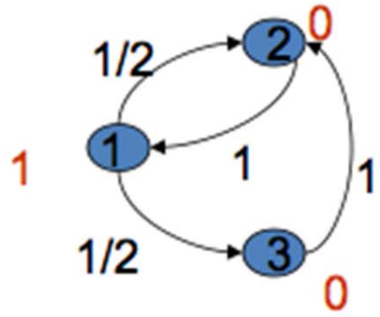
Convergence in DeGroot model



$$T = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$b(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

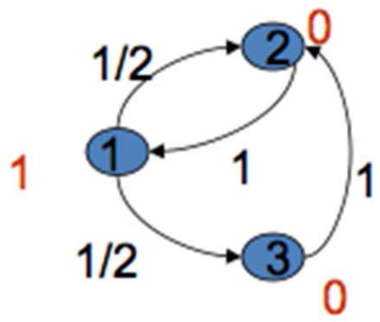
Convergence in DeGroot model



$$T = \begin{pmatrix} 0 & ½ & ½ \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$b(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad b(1) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Convergence in DeGroot model



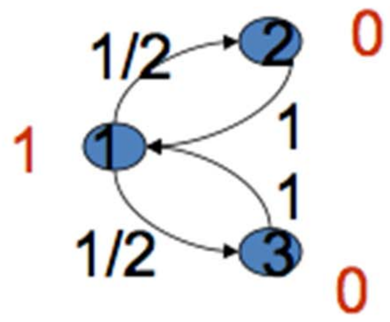
$$T = \begin{pmatrix} 0 & ½ & ½ \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$b(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad b(1) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1/2 \\ 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 3/4 \\ 1/2 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1/4 \\ 3/4 \\ 1/2 \end{pmatrix} \cdots \rightarrow \begin{pmatrix} 2/5 \\ 2/5 \\ 2/5 \end{pmatrix}$$

- Beliefs converge over time and agents reach a consensus.

Convergence in DeGroot model: failure to converge

- It is possible that the updating process **fails to converge**, as in the following example.

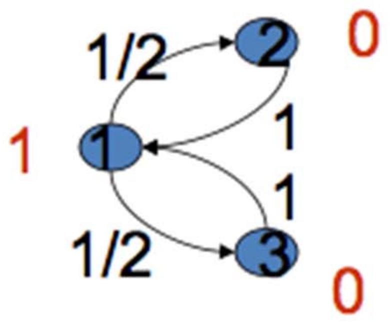


$$T = \begin{pmatrix} 0 & ½ & ½ \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$b(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

(Jackson, M.O., 2013)

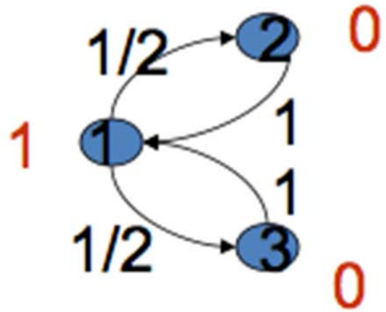
Convergence in DeGroot model: failure to converge



$$T=\begin{pmatrix} 0 & ½ & ½ \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$b(0)=\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad b(1)=\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

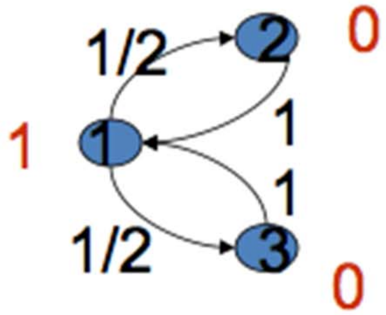
Convergence in DeGroot model: failure to converge



$$T = \begin{pmatrix} 0 & ½ & ½ \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$b(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad b(1) = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

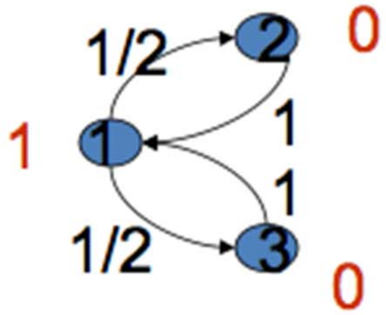
Convergence in DeGroot model: failure to converge



$$T=\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$b(0)=\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad b(1)=\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \ldots \rightarrow$$

Convergence in DeGroot model: failure to converge



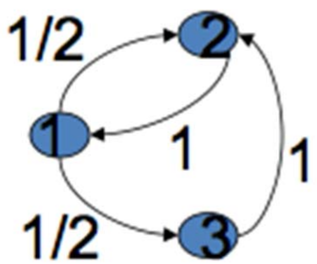
$$T = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$b(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad b(1) = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \ldots \rightarrow$$

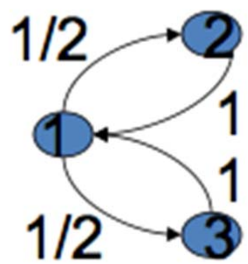
- There are **directed cycles** in the network and all the cycles are of lengths that are multiples of 2.
- Matrix T is **periodic**, allowing the process to cycle without converging.

Convergence in the DeGroot model

- The necessary and sufficient condition for convergence of T is that **T is aperiodic**.
- T is **aperiodic** if the greatest common divisor of its cycle lengths is one.
- Almost any (sufficiently large) "real" society will be aperiodic.
    - So, **do all societies converge**?
    - If yes, **how fast and to what beliefs**?



aperiodic



periodic

DeGroot Opinion Formation Model Example

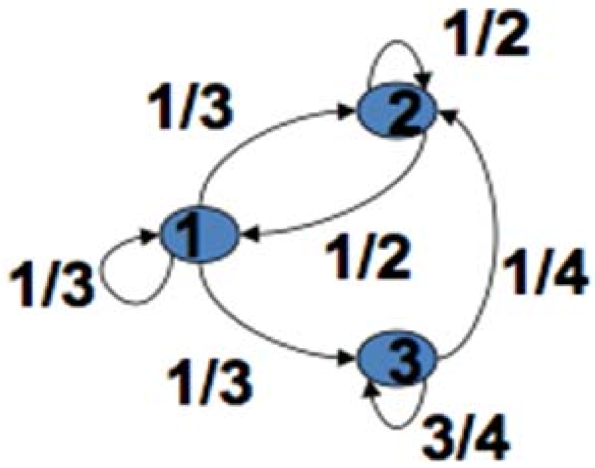
Illustrative Example

- Suppose that we have three national central banks:
  - **Deutsche Bundesbank**
  - **Banque de France**
  - **Bank of Greece**.

- Deutsche Bundesbank is run by **Jens Weidmann**.
- Banque de France is run by **François Villeroy de Galhau**
- Bank of Greece is run by **Yannis Stournaras**

DeGroot Opinion Formation Model Example

- The direct influence of these three governors on each other is captured by the following network diagram:



DeGroot Opinion Formation Model Example

- The social interaction matrix T indicates that:
  - Deutsche Bundesbank puts equal weight (**1/3** each) on Banque de France and Bank of Greece.
  - Banque de France weights its own beliefs slightly more, but completely discounts Bank of Greece.
  - Bank of Greece puts the most weight/trust in its own beliefs (**3/4**), puts a weight of **1/4** on Banque de France and (perhaps unwisely…) completely disregards Deutsche Bundesbank.

DeGroot Opinion Formation Model Example

- Now, suppose that the three central bankers debate on whether a future event (say, Grexit) is possible:
  - Deutsche Bundesbank initially believes that this event is a certainty, so its **p(0)=1**.
  - Banque de France and Bank of Greece start off believing that there is no chance of this event happening, so their **p(0)=0**.

- Let's see how beliefs change over time due to the deliberations that take place between the three bankers.

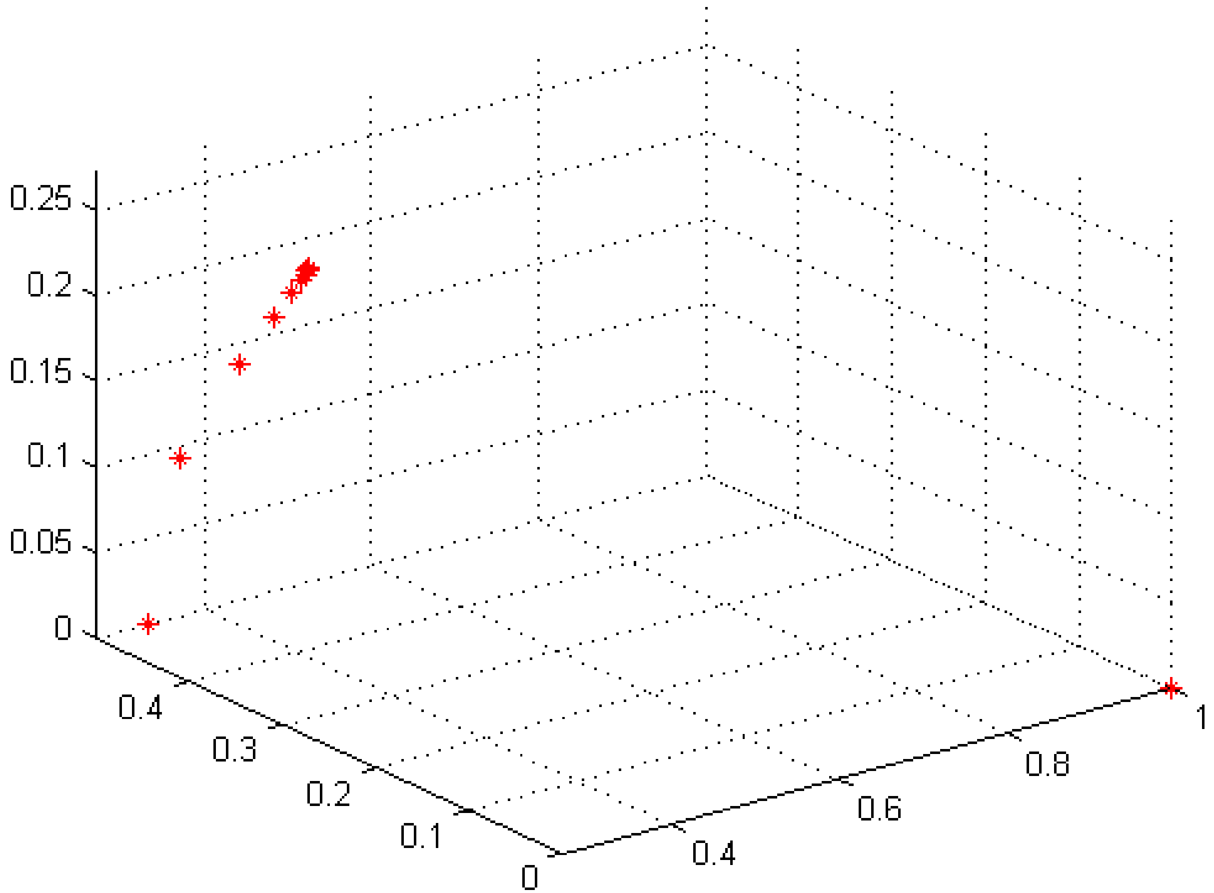
DeGroot Opinion Formation Model Example

- By running a computer simulation for the previous network, given the initial vector of beliefs we get **convergence** and **consensus** after (approximately) 8 rounds of belief updating:

Time	Bank1	Bank 2	Bank2
0	1	0	0
1	0,333333	0,5	0
2	0,277778	0,416667	0,125
3	0,273148	0,347222	0,197917
4	0,272762	0,310185	0,235243
5	0,27273	0,291474	0,253979
6	0,272728	0,282102	0,263352
7	0,272727	0,277415	0,26804
8	0,272727	0,275071	0,270384
9	0,272727	0,273899	0,271555
10	0,272727	0,273313	0,272141
11	0,272727	0,27302	0,272434
12	0,272727	0,272874	0,272581
13	0,272727	0,272801	0,272654
14	0,272727	0,272764	0,272691

DeGroot Opinion Formation Model Example

- The trajectory of the bankers' beliefs may also be represented by the following 3-dimensional plot.



DeGroot Opinion Formation Model Example

- The same result may also be obtained analytically based on the following facts:
  - T is **strongly connected** since there exists a direct path connecting any pair of bankers in the network.
  - T is **aperiodic** since there exist no cycles of beliefs and no feedback loops where beliefs flow from one banker through all other bankers and then end up influencing the initial banker again.
- Therefore, the limiting vector of beliefs will be given by $\boldsymbol{p}(\infty) = \begin{bmatrix} \frac{\mathbf{3}}{\mathbf{11}} & \frac{\mathbf{3}}{\mathbf{11}} & \frac{\mathbf{3}}{\mathbf{11}} \end{bmatrix}$

DeGroot Opinion Formation Model Example

- The limiting social influence matrix will be given by:

$$\boldsymbol{T}(\infty) = \begin{bmatrix} \frac{\mathbf{3}}{\mathbf{11}} & \frac{\mathbf{4}}{\mathbf{11}} & \frac{\mathbf{4}}{\mathbf{11}} \\ \frac{\mathbf{3}}{\mathbf{11}} & \frac{\mathbf{4}}{\mathbf{11}} & \frac{\mathbf{4}}{\mathbf{11}} \\ \frac{\mathbf{3}}{\mathbf{11}} & \frac{\mathbf{4}}{\mathbf{11}} & \frac{\mathbf{4}}{\mathbf{11}} \end{bmatrix}$$

T^{∞} values show the **relative influence** of each node in the network:

- Agents 2 and 3 have a 36% relative influence
- Agent 1 has a 27% relative influence

DeGroot Opinion Formation Model Example

Conclusions / Remarks:

- The networked "learned", i.e. converged to a common belief.
- In the absence of additional information, we cannot know if the common belief is **accurate** (i.e. does the event really have a 27% probability of occurring?)
- Not all members of the society learned the common belief **at the same time** (Deutsche Bank learned it after only 3 steps).
- Not all members of the society have the same influence (the relative influence of Banque de France and Bank of Greece is the same while the relative influence of Deutsche Bank is slightly less than the influence of the other banks).

Social Influence

Measuring influence in the De Groot Model

Influence in Krackhardt's advice network

Golub and Jackson (2010): Social Influence in the DeGroot model

Measuring Influence

- Where do rows of T^t converge?
- Seek a **row vector s**, indicating relative influence (limit belief is **s·p(0)**)
- **s·p(0) = s·T·p(0),** therefore **s = s·T** – i.e. **s is the left unit eigenvector of T**.

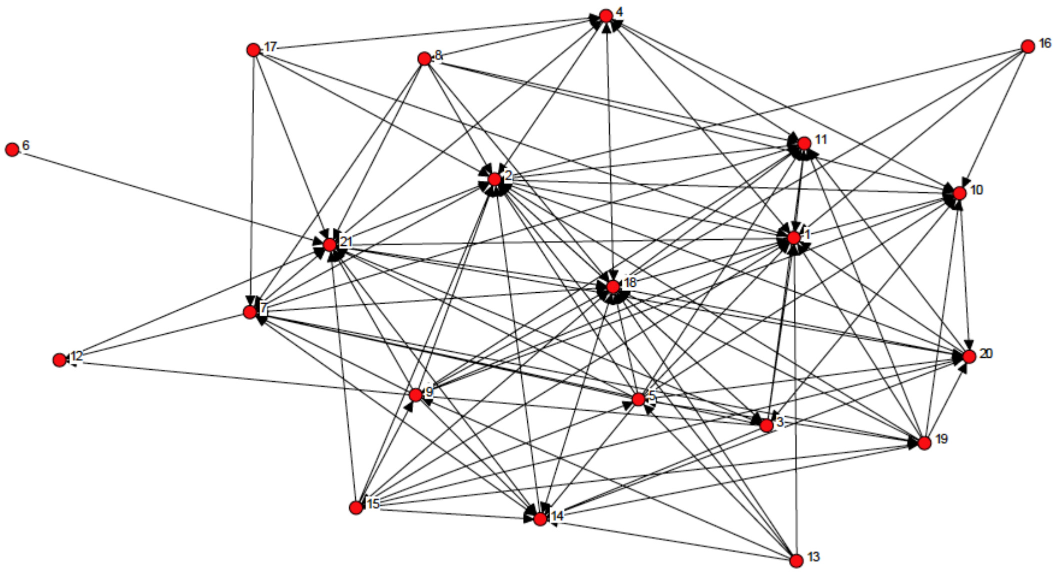
Detecting influentials

- Since **s** corresponds to a (left-hand unit) eigenvector of T: $s_j = \sum_i T_{ij} s_i$
- **High influence comes from being paid attention to by people with high influence**
- This measure of influence is related to **eigenvector centrality**.

Example: Influence in Krackhardt's Advice Network

Example: Influence in Krackhardt's Advice Network

- Data for a small manufacturing firm
- 100 employees and 21 managers
- Information from managers: who do they consult with, seek advise etc.

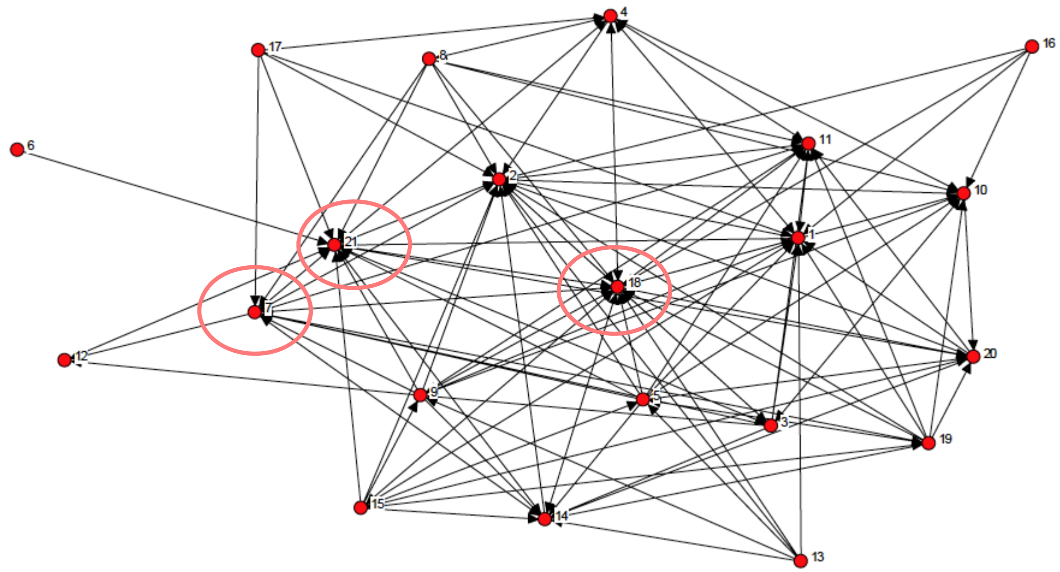


Krackhardt, D. (1987) Cognitive Social Structures, *Social Networks*, 9, pp. 109-134.

Example: Influence in Krackhardt's Advice Network

Example: Influence in Krackhardt's Advice Network

- Develop a T matrix (advice matrix) by **normalizing** each row (sum to 1): how a given agent weights others' opinions
- If i seeks advice from 7 different agents including agent j, then $T_{ij} = 1/7$ (**not weighted information**).
- **Calculate the s vector** directly as the left-hand unit eigenvector.



Example: Influence in Krackhardt's Advice Network

Example: Influence in Krackhardt's Advice Network

- **No agents seek advice from 6, 13, 16 and 17**; they are outside of the single closed strongly connected group
- Influence can be much different from indegree, e.g., 21 has the highest influence, even though agent 18 advises more agents. Two reasons:
  a) influence is higher when an agent is paid attention to by other agents who are in turn paid attention to more
  b) one gets more influence when advising agents who seek advice from relatively fewer agents. e.g., agent 7 (head of firm) has substantial influence even though advises only 6 others.

Note: agent 7 advises 2, 18, and 21, all influentials (and at the second level of hierarchy).

label	s	level	dept.	age	tenure
1	0.048	3	4	33	9.3
2	0.132	2	4	42	19.6
3	0.039	3	2	40	12.8
4	0.052	3	4	33	7.5
5	0.002	3	2	32	3.3
6	0.000	3	1	59	28
7	0.143	1	0	55	30
8	0.007	3	1	34	11.3
9	0.015	3	2	62	5.4
10	0.024	3	3	37	9.3
11	0.053	3	3	46	27
12	0.051	3	1	34	8.9
13	0.000	3	2	48	0.3
14	0.071	2	2	43	10.4
15	0.015	3	2	40	8.4
16	0.000	3	4	27	4.7
17	0.000	3	1	30	12.4
18	0.106	2	3	33	9.1
19	0.002	3	2	32	4.8
20	0.041	3	2	38	11.7
21	0.201	2	1	36	12.5

Example: Influence in Krackhardt's Advice Network

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15	0.015	3	2	40	8.4
16	0.000	3	4	27	4.7
17	0.000	3	1	30	12.4
18	0.106	2	3	33	9.1
19	0.002	3	2	32	4.8
20	0.041	3	2	38	11.7
21	0.201	2	1	36	12.5

Golub and Jackson (2010): Social Influence in DeGroot model

Example: Influence in Krackhardt's Advice Network

- No agents seek advice **from 6, 13, 16 and 17;** they are outside of the single closed strongly connected group
- Influence can be much different from indegree, e.g., 21 has the highest influence, even though agent 18 advises more agents. Two reasons:
  a) influence is higher when an agent is paid attention to by other agents who are in turn paid attention to more
  b) one gets more influence when advising agents who seek advice from relatively fewer agents. e.g., **agent 7** (head of firm**) has substantial influence even though advises only 6 others.**

Note: agent 7 advises 2, 18, and 21, all influentials (and at the second level of hierarchy).

label	s	level	dept.	age	tenure
1	0.048	3	4	33	9.3
2	0.132	2	4	42	19.6
3	0.039	3	2	40	12.8
4	0.052	3	4	33	7.5
5	0.002	3	2	32	3.3
6	0.000	3	1	59	28
7	0.143	1	0	55	30
8	0.007	3	1	34	11.3
9	0.015	3	2	62	5.4
10	0.024	3	3	37	9.3
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Rate and Accuracy of Convergence

Rate of convergence and segregation

Accuracy of convergence: The wisdom of crowds

Rate of convergence and segregation

How quickly do beliefs reach their limit?

- a few rounds of updating?
- does new information enter the system over time?

If convergence is slow, then there may be persistent heterogeneous beliefs in a society even though it might tend towards a consensus.

Question: What determines the rate (speed) of convergence?

Rate of convergence and segregation

Suppose two agents:

- **If they hold similar beliefs, fast convergence**.
- **If they hold very different beliefs**, difference will reign for long: **slow convergence.**
  - e.g., each agent weights own opinion heavily and disregards the other.
  - **Rate of convergence depends on how much T_{11}** (weight that 1 places on 1) **differs from T_{21}** (weight that 2 places on 1).

- For the general (multi-agent) case, **the rate of convergence is related to the difference in the weights** that different agents place on one another – i.e. to the level of **segregation** or **homophily** in the network.

- **"Real" worlds, with strong homophily/segregation, will take long to converge**.

Accuracy of convergence and wise crowds

- In a connected network, people reach asymptotic consensus.

Are consensus beliefs 'correct'?

- Maybe the shared information concerns something that is objectively measurable, e.g. the reliability of a product.
- In such instances, during updating and convergence, the question is **whether beliefs converge to the right probability** (expectation etc.).
- This question is examined in detail by **Golub and Jackson (2010).**

Accuracy of convergence and wise crowds: Golub and Jackson (2010)

General idea of the model

- Agents initially receive a noisy signal about the true state of the world.
- They update their beliefs as a weighted average of the neighbor's beliefs.

How does the **accuracy of limit consensus belief** depend on:

- **network structure**?
- **influence**?

- In general, **if no agent is overly influential** then the consensus value converges to the true state of the world in probability, i.e., **everybody learns the true state**.

Golub, B. and Jackson M.O. (2010). Naïve Learning in Social Networks and the Wisdom of Crowds, *American Economic Journal*: Microeconomics, 2(1), 112-49.

Accuracy of convergence and wise crowds: Golub and Jackson (2010)

Modeling Framework

Uncertainty Structure

- Suppose **true state** is **μ**
- Agent i sees **$p_i(0) = \mu + \varepsilon_i$**
- **ε_i** has **0 mean** and **finite variance**, bounded below and above.
- The distribution of signals can differ across agents but **signals are independent** conditional on μ.

Accuracy of convergence and wise crowds: Golub and Jackson (2010)

Wise Crowds

- Suppose we have a large society.
- If agents pooled their information, would they have an accurate estimate of μ?
- For sequences of societies indexed by n, does

$$\text{Prob} \lim_{t\to\infty}\left[\left|p_j^n(t)-\mu\right|>\delta\right]\to 0 \text{ as } n\to\infty \text{ for all } \delta, j \text{ ?}$$

A weak law of Large Numbers

- Suppose ε_i 's independent, zero mean, with finite variance (bounded from below). Then, the society is wise if and only if:

$$\max_i s_i^n \to 0 \text{ as } n\to\infty$$

Wise crowds if and only the influence of the most influential agent vanishes in the limit (i.e. if and only if no agent retains too much influence in the limit).

Key takeaways

- **Naïve Learning**
  - Agents learn by repeatedly averaging their neighbors' beliefs
- **Societies converge if the social influence matrix is aperiodic**
  - It will almost always be in strongly connected social group of large size
  - Segregation/homophily will slow the rate of convergence
- **The relative influence of agents determines whether society will 'learn' (i.e. converge to the 'right' belief)**
  - Reciprocity increases the chances of social learning
  - Overly influential agents imply potentially wrong convergence (herding)