

Τριγωνομετρία

Βασικές ιδιότητες: $\cos(-x) = \cos x$, $\sin(-x) = -\sin x$,

$$\cos^2 x + \sin^2 x = 1, \quad 1 + \operatorname{tg}^2 x = 1/\cos^2 x,$$

$$n \in \mathbb{N} \Rightarrow \cos(n\pi) = (-1)^n, \quad \sin((2n-1)\pi/2) = (-1)^{n-1}$$

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin x$	0	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos x$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2	0

Βασικές τιμές:

Βασικές ταυτότητες:

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1 = \frac{1-\operatorname{tg}^2 x}{1+\operatorname{tg}^2 x}$$

$$\sin 2x = 2\cos x \sin x = \frac{2\operatorname{tg} x}{1+\operatorname{tg}^2 x}$$

$$2\cos x \cos y = \cos(x-y) + \cos(x+y)$$

$$2\sin x \sin y = \cos(x-y) - \cos(x+y)$$

$$2\sin x \cos y = \sin(x+y) + \sin(x-y)$$

$$\sin x + \sin y = 2\sin \frac{x+y}{2} \cos \frac{x-y}{2}, \quad \sin x - \sin y = 2\cos \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\cos x + \cos y = 2\cos \frac{x+y}{2} \cos \frac{x-y}{2}, \quad \cos y - \cos x = 2\sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

Βασικές δυναμοσειρές - σειρές Maclaurin

$$\bullet \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad x \in (-1, 1).$$

$$\bullet e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad x \in \mathbb{R}.$$

$$\bullet (1+x)^r = \sum_{n=0}^{\infty} \binom{r}{n} x^n = \sum_{n=0}^{\infty} \frac{r(r-1)\cdots(r-n+1)}{n!} x^n, \quad x \in (-1, 1), \quad r \in \mathbb{R}.$$

$$\bullet \ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n, \quad x \in (-1, 1].$$

$$\bullet \sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}, \quad \cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}, \quad x \in \mathbb{R}.$$

$$\bullet \sinh x = \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} x^{2n+1}, \quad \cosh x = \sum_{n=0}^{\infty} \frac{1}{(2n)!} x^{2n}, \quad x \in \mathbb{R}.$$